# KĀTYĀYANA ŚULBA SŪTRA

*(With English Translation, Explanatory Notes, Diagrams and Articles)*

*Edited by*

**S. D. Khadilkar**

VAIDIKA SAMŚODHANA MAṆḌALA

*2003*

टिळक-महाराष्ट्र-विद्यापीठ-शाखाभूत-

**वैदिक-संशोधन-मण्डलेन**

**प्रकाशितम्**

# कात्यायन-शुल्बसूत्रम्

( आङ्ग्लानुवादेन टिप्पणीभिश्च समलंकृतम् )

**श्री. दा. खाडिलकर**

इत्यनेन संपादितम्

**शाके १९२५**

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# KĀTYĀYANA ŚULBA SŪTRA

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*Edited by*

**S. D. Khadilkar**

VAIDIKA SAMŚODHANA MAṆḌALA
POONA

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# SECTION I

## TEXT TRANSLATION AND NOTES

THIS BOOK IS DEDICATED

TO

THE MEMORY OF

**My Mother and Father**

—Editor

# FOREWORD

His Holiness Sri Sri Bharathi Theertha Mahaswamiji.

I take this opportunity to offer my heartfelt gratitude at the holy feet of His **Holiness Sri Sri Bharathi Theertha Mahaswamiji.**

This Katyayana Sulba Sutra (text with English Translation) was not available for past few years. We could bring out this second edition because of the substantial financial help which we received timely from **Sanganur Subramania Iyer Trust, Coimbatore - Sole Trustee : Sri Sringeri Math - as per the direction of His Holiness Jagadguru Shankaracharya Mahasamsthanam Dakshinamnaya Sri Sharada Peetham, Sri Sri Bharathi Theertha Mahaswamiji.**

I acknowledge the sincere efforts taken by Ms. Vaijayanti Dhole-Patil and Ms. Surekha Wagle, Step in Services, Pune - 11, for perfect and prompt printing of this book.

June 2003

Dr. Bhagyalata Pataskar
Secretary
Vaidika Samshodhana Mandala,
Pune 37

# PREFACE

Long before Dr. C. G. Kashikar suggested to me to take up the study of the Śulba Sūtras, when I had an opportunity to discuss with him certain problems relating to the Vedic sacrificial system. In course of time I prepared the Marathi translation of the Kātyāyana Śulba Sūtra with the help of Dr. M. R. Bondre from Baroda. Thereupon I revised it with the help of Prof. Patwardhan of Dapoli, Later on, it was examined by Dr. Kashikar. But I thought it desirable to prepare an English Version of the same and the present work is the outcome of my studies.

Shri Dixit, Shri Haribhau Desai and Shri Pade Śāstrī, all from Baroda helped me in solving my difficulties. In compiling this Monograph, many people helped me in various other ways. I am thankful to all of them. I must also thank, the Executive Council of the Vaidika Saṁśodhana Maṇḍala, Poona, for publishing this dissertation.

My hearty thanks are due to Shri N. S. Sontakke for undertaking this publication.

My sincere thanks are also due to Dr. T. N. Dharmadhikari who bore a large share of the burden in the matter of the printing of this volume.

I take this opportunity to express my heartfelt thanks to the proprietors of the Veda-Vidyā Mudraṇālaya who carried out the intricate printing of this volume with great care.

Akṣayya Tṛtīyā
Śake 1896
25 April 1274

S. D. Khadilkar

# INTRODUCTION

In this small Introduction I intend to deal with the following topics: (1) What is meant by *Śulba Sūtras*? (2) Their short history (3) Contents of each *Śulba Sūtra*. (4) Contents of *Kātyāyana Śulba Sūtra* in detail. (5) General remarks. (6) Important items requiring consideration, and (7) Conclusions.

## (1) The Śulba Sūtras.

The *Śulba Sūtras*, which form a part of the Sūtra literature are intended to lay down the rules of demarcation of the various sacrificial altars, pandals, and places for sacred fire etc.. provided as per directions quoted by the authors of *Saṁhitās Brāhmaṇas* and *Śrauta Sūtras*, for performing the various sacrifices, both *Nitya and Kāmya*.

It should be noted that the procedure to be followed while performing various Vedic sacrifices is scattered over the *Brāhmaṇa* and *Saṁhitā* literature of *Yajurveda*. But the rules pertaining to the Mathematical side of it, especially the Geometrical portion involved, which is required for constructing the various forms of altars, which are either a square, a triangle, a trapezium or a circle in shape and the construction of various *citis*, which have the shape of birds, with spread out wings and tail etc. are given only in the *Śulba Sūtras*.

The *Śulba Sūtras* generally occur as one or more chapters of the *Kalpa* or *Śrauta Sūtras* or are separate. The system of measurement of the various sacrificial places was being carried out by means of '*Śulba*' or a measuring cord and the word '*Sūtra*' denotes rules in concise and short forms and therefore, these are termed as *Śulba Sūtras*.

The duty of measuring and preparing the various sacrificial places was, assigned to *Adhvaryu* and his assistants,who belonged to the *Yajurveda*, which abounds different schools. These schools are represented by different *Śrauta Sūtras*. So each of the *Śrauta Sūtra* belonging to *Yajurveda* was entitled to have its own *Śulba Sūtra*.

## (2) Their Short History :

Only the following *Śulba Sūtras* are available at present. These are : (1) *Baudhāyana*, (2) *Āpastamba*, (3) *Satyāṣāḍha*, (4) *Mānava*, (5) *Maitrāyaṇīya* (6) *Vārāha* and (7) *Vādhūla*. All these belong to *Kṛṣṇa Yajurveda*. Only the *Sūtra* of *Kātyāyana* belongs to *Śukla Yajurveda*.

Out of these (1) *Baudhāyana* (2) *Āpastamba* and (3) *Kātyāyana* deal with the Mathematical portion—namely Geometry, in detail : (4) *Mānava* gives a very scanty description of the Geometry. It deals only with the square and circle. The (1) *Baudhāyana*, (2) *Āpastamba* and (3) *Mānava* make use of these rules in the construction of various *citis*. But the description gives only their dimensions according to their shapes.

The most important points observed in all of them are :—

(1) The shapes of the *citis* or the altars of the sacred fires may differ in shape but their areas will remain the same.

(2) In the construction of *citis*, there will be five layers and each layer will consist of 200 bricks.

(b) In the case of the three sacred fires only *Gārhapatya* will have five layers and each layer will consist of 21 bricks.

## 3) Contents of each Śulba Sūtra :

### (A) The Baudhāyana Śulba Sūtra :

This is divided into three parts. (1) The first part is devoted to the Geometrical portion,required in the construction of various *vedis*, *citis* and pandals etc. In addition to this, the distance between the sacred fires, their shapes, the forms of different bricks and their numbers and how to place them is described Then follows the description of various *Dhiṣṇyas*.

(2) The second and third chapters deal with *vedis*, *citis*, pandals etc.giving their sizes, shapes of bricks required and how and where to place them.

(3) *Dvārakānātha Yajvā* and *Veṅkateśvara Dīkṣita* are the commentators.

(4) The text along with the extracts of the commentary by Shri *Dvārkānātha Yajvā* with its translation in English was published, in the *Pandit*, a magazine devoted to Sanskrit language; and issued from Benaras by Dr. G. Thibaut in the year 1874-76.

The text only is published in the *Baudhāyana Śrauta Sūtra* edited by W. Caland.

The *Baudhāyana Śulba Śūtra*, with translation in English edited by Dr. G. Thibaut was republished by Dr, *Satya Prakāśa* and *Rām Svarūpa* from Delhi in 1968.

In the first chapter, the first 20 *sūtras* give the definitions of different measures which occur in the *Sūtras*.

The next 20 *Sūtras* gives us an idea of four different methods of constructing a square. *Sūtra* 45 gives a clear definition of the square on the diagonal of a square. *Sūtra* 48 gives the definition of the square on the diagonal of a rectangle with reference to two other sides (this proposition is at present named after Pythagoras) and *Sūtra* 68 gives the value of $\sqrt{2}$.

### (B) Āpastamba Śulba Sūtra :

(a) (1) This *sūtra* is divided into 6 chapters, which are again sub-divided into 21 *khaṇḍas*.

(2) The first 3 *khaṇḍas* deal mainly with the Geometrical rules.

(3) The remaining portion consists of the description of the various *vedis*; *citis* etc., and forms and sizes of bricks required for their construction, with instructions as to how and where to place them.

(b) The Commentators of this *Śulba sūtra* are : (1) *Kapardisvāmī* (2) *Karvindasvāmī* and (3) *Sundararāja*.

(c) The commentaries are most useful. At certain places, improvements are suggested by the commentators in the rules given by the author of *Śulba sūtra*.

(d) This *Śulba Sūtra* was first edited and published with a German translation by A. Burk in Germany in the year 1902.

(e) The Sanskrit text with commentaries is published by the University of Mysore as a Sanskrit series No. 73 in the year 1931.

(f) An English translation along with the text is edited and published by Dr. *Satya Prakāśa* and *Rāma Svarūpa* from Delhi in 1968.

**(C) Satyāṣāḍha Śulba Sūtra :**

This *Śulba Sūtra* is completely identical with *Āpastamba* published by the University of Mysore.

(a) Its commentator is Shri *Mahādeva Dīkṣita, Somayājī*.

(b) The text is published with comments by *Ānandāśrama* Poona as a part of the *Satyāṣāḍha Śrauta Sūtra* in the year 1930. (Sanskrit series No. 53).

(C) Geometrical rules ;

(1) Rule 8 gives the definition of the theorem of Pythagoras in the case of a rectangle.

(2) Rule 10 gives the definition of the hypotenuse in the case of a square (*Dvikaraṇī*) followed by the value of $\sqrt{2}$.

**(D) Mānava Śulba Sūtra :**

(1) The text is published in the *Mānava Śrauta Sūtra* edited & translated by a Dutch Lady Dr. J. M, Van Gelder (*Śatapiṭaka* series Volumes 17 and 27).

(2) It consists of three parts :

(a) The first part consists of the East-West line; the relative positions of the three sacred fires etc.

(b) The second part is called *Uttareṣṭaka* and the third is *Vaiṣṇava*.

(c) This deals mainly with the square and the circle and the description of few *citis*.

(d) Its commentator is *Śankara*.

**(E) Maitrāyaṇīya Śulba Sūtra :**

This is still a manuscript and is available with the Asiatic Society of Bombay and the Vaidika Samśodhana Maṇḍala, Poona.

(b) The Sanskrit text consists of corrupt Sanskrit language. The same will have to be corrected and finally read, so as to enable one to undertand what it contains.

(c) At the end, the author says that it is a *Bhāṣya* and not a *Śulba Sūtra*.

**(F) Vāraha Śulba Sūtra :**

(a) This manuscript is available with the Oriental library of Mysore.

**(G) Vādhūla Śulba Sūtra :**

(a) The manuscript of the *Vādhūla Śrauta Sūtra* is available at Madras but it will have to be ascertained whether it contains a *Śulba Sūtra*.

**(H) Kātyāyana Śulba Sūtra :**

(1) This *Śulba Sūtra*, just like *Mānava*, consists of both prose and verse.

(2) There are 6 chapters consisting of 101 *Sūtras* and the 7th chapter consists of 39 verses.

All these chapters give the important rules of Geometry.

(3) (a) The text of this *Śulba Sūtra* with commentaries of *Rāma* and *Vidyādhara Śarmā* was published by the *Acyuta Granthamālā* from Benaras in 1928 No. 3.

(b) The same text with commentaries from *Karka* and *Mahīdhara* was published by the Haridas Sanskrit Granthamālā—The Choukhamba Sanskrit series from Banaras in 1236, No. 120.

(c) The rules only of the first two chapters of this *Sūtra* were translated in English and then published in the Magazine *Pandit* by Dr. G. Thibaut from Benaras in 1882.

(d) The enunciation of the Pythagoras Theorem is given in *Sūtra* II pertaining to a rectangle. *Sūtra* 12 deals with the enunciation of the value of the hypotenuse, in the case of the square, *Dvikaraṇī* and the *Sūtra* 13 gives the value of $\sqrt{2}$. All these rules are from chapter 2.

The subject matter of all the *Śulba Sūtras* is practically the same—namely, the construction of *vedis, citis*, pandals etc. Naturally therefore 60% of the Geometrical rules followed by them are the same. As I am dealing with the *Kātyāyana Śulba Sūtra*, I intend to give a list of *Sūtras*, occuring therein serially, so as to enable one to appreciate the progress of Geometry made in those days.

### Chapter I:

| Sr. No. | Subject described in the sūtra | Sūtra No. |
|---|---|---|
| 1 | How to construct a *vedi*, *citi* etc. of a given shape, with the help of a rope will be described in this *Śulba Sūtra*. | 1 |
| 2 | How to locate the East-West line. | 2 |
| 3 | How to locate the North-South line. | 3 |
| 4 | Provide slings at both ends of the cord. | 4 |
| 5 | Provision of special signs on the cord is necessary for measuring purposes. | 5 |
| 6 | Fix pegs at the end of the East-west line. | 6 |
| 7 | Fix pegs on the corners of *Śroṇi* and *Aṁsa*. | 7 |
| 8 | How to construct a square. | 8 to 10 |
| 9 | Everywhere the use of this method should be made. | 11 |
| 10 | Construction of right angled triangles with the help of the *Nirañchana* Mark. | 12 to 15 |
| 11 | The position of the signs of *Śroṇī* and *Aṁsa* in the case of a square, a rectangle and a triangle. | 16 to 18 |
| 12 | With the help of the above, how to construct pandals such as *Prāgvaṁśa*; *Śālā*; and *Sadas* etc. is explained. | 19 to 22 |
| 13 | Definition of the word *Aparimita*. | 23 |
| 14 | Measure of the Increase or Decrease. | 24 to 25 |
| 15 | Location of the place of *Dakṣiṇāgni*. | 26, 27, & 29 |
| 16 | Location of *Utkara*. | 28 & 30 |

### Chapter II:

| | | |
|---|---|---|
| 1 | Measures like *Aṅgula; Iṣā; Akṣa* and *Yuga* etc., used in the construction of a *vedi*, having the shape of a chariot are given in these *Sūtras*. | 1 to 5 |
| 2 | *Paitṛki Vedi* (a *vedi*, having a square shape, whose corners are turned towards the cardinal directions and whose area is 1 square *puruṣa*). | 6 |
| 3 | Five types of measuring cords, (with their names) used for measuring purposes are given in this *sūtra*. | 7 |
| 4 | *Uttarā Vedis* are constructed with the help of measures such as $\sqrt{10}$ and $\sqrt{40}$, derived with the help of a rt. ∠ ed △. | 8, 9 & 10 |
| 5 | The square on the diagonal of a rectangle is equal to the sum of the squares on the other two sides (this is in other words the theorem of Pythagoras). | 11 |
| 6 | In the case of a square, the square on the diagonal of a square is double that of the square on its side. This is the definition of $\sqrt{2}$. | 12 |
| 7 | Value of $\sqrt{2}$. | 13 |
| 8 | *Trikaraṇī* : How to construct a square, whose area will be three times that of the original square. | 14 |
| 9 | How to costruct a square, whose area will be 1/3rd of the original square (divide the square into 9 equal parts etc. All this is necessary for the *Prakarma* measure of the *Sautrāmaṇī Vedi*. | 15 to 20 |
| 10 | The addition of two equal squares. | 21 |
| 11 | The addition of two unequal squares. | 22 |

### Chapter III:

| | | |
|---|---|---|
| 1 | To construct a square equal in area to the difference of two squares | 1 |
| 2 | To construct a square equal in area to a rectangle. | 2 to 3 |
| 3 | To construct a rectangle equal in area to that of a square. | 4 |
| 4 | If no instructions are provided, use the measure stated to construct a square. | 5 |
| 5 | Method to find the area of a square or a rectangle, when the measure is in integers, the area increases. When the measure is a fraction of the original measure the area decreases. How to reduce this area is explained before. The increase or decrease in area depends on the increase or decrease in the measure. | 6 to 12 |
| 6 | To construct a circle equal in area to that of a square. | 13 |
| 7 | To construct a square equal in area to that of a circle. | 14 |

### Chapter IV:

| | | |
|---|---|---|
| 1 | Names of different *citis*. | 1 |
| 2 | Some information regarding *Droṇa* and *Rathacakra citi*. | 2 to 4 |
| 3 | To construct a triangle equal in area to that of a square. | 5 |
| 4 | To construct an *Ubhayataḥ Prauga* (Two isosceles triangles joined at the base) equal in area to that of a rectangle. | 6 |
| 5 | To construct a square equal in area to that of a triangle. | 7 |
| 6 | To construct a square equal in area to that of an *Ubhayataḥ Praugha*. | 8 |
| 7 | To construct a squure equal in area to that of three or five triangles or triangles having equal or unequal angles. | 9 to 12 |

### Chapter V:

| | | |
|---|---|---|
| 1 | Increase the area of an *Agni* from seven-fold to 101-fold by an increase of one *sq. puruṣa*. | 1 |
| 2 | The *Vedis* to be constructed for an *Aśvamedha citi*. How to construct them. | 2 and 3 |
| 3 | How to increase the measure by *Puruṣa*. | 4 |

| | | |
|---|---|---|
| 4 | Three different methods are adopted to achieve the same. | 5, 7, 8 & 10 |
| 5 | Sizes of *Bṛhatī* and *Padamātrī* square bricks | 6 |
| 6 | Definition of the measure of a *Puruṣa* and *Pada.* | 9 |
| 7 | The measure of a *puruṣa* is increased, without the addition of the measures of *aratni* and *vitasti.* | 11 |

CHAPTER VI :

| | | |
|---|---|---|
| 1 | Along with the increase in the measure of a *Puruṣa,* increase in the size of the *vedi* and bricks is suggested. | 1 |
| 2 | The measure of a *Prakarma* for the 101-fold *Agni. Prakarma* measure is also to be increased, like the increase in the *Puruṣa* measare already suggested. | 2 to 5 |
| 3 | No increase is to be made in the sizes of *Dakṣiṇāgni* and *Gārhapatya.* | 6 |
| 4 | To construct a square equal in area to a number of similar equal squares. | 7 |
| 5 | Increase in the *vedi,* having eleven posts. The same is called *Śikhaṇḍinī Vedi,* | 8 to 13 |

CHAPTER VII : Verses

| | | |
|---|---|---|
| 1 | Information regarding the signs to be adopted on a rope and how to make use of them in constructing a *Paurṇa-māsikī vedi.* | 1 |
| 2 | What knowledge is necessary for a man conversant with *Śulba.* | 2 to 5 |
| 3 | What are the important requisits of a wooden peg and a wooden hammer required to drive them. | 6 to 10 |
| 4 | The qualities of a measuring cord and how to prepare the same. | 11 to 13 |
| 5 | Signs to be provided on the measuring cord of the *Sadas* hall and how to use the same. | 14 |
| 6 | Measure of the cord of a *Saumikī vedi.* | 15 |
| 7 | Measure of the cord of a *Sautrāmaṇī vedi.* | 16 |
| 8 | If due to fog or clouds, the stars and the light is not visible, make use of a lamp. | 17 |
| 9 | Those who conduct the sacrifice, must acquire the thorough knowledge of the theorem and constructions to be used according to the principles laid down by the *Śulba* and then successfully carry out the sacrifice. | 18 |
| 10 | Care is taken to see that the measure is neither decreased nor increased and is pleasing to the eye. | 19 |
| 11 | Measures in constructing the altar to be followed. | 20 & 21 |
| 12 | Measures of *aṅgula, aratni* and *hasta* etc. | 22;& 25 |
| 13 | No. of golden coins to be used. | 26 to 29 |
| 14 | The size of the brick decreases while drying and burning. Increase the size to make up the loss. | 3 |

| | | |
|---|---|---|
| 15 | Method of constructing a rt. ∠ed triangle. | 32-35 |
| 16 | The method of fixing the East-West line. | 35 |
| 17 | The measure of *Prakarma* expressed in *aṅgulas.* | 34 |
| 18 | Measures used in the construction of the altars of the three sacred fires. | 37 & 38 |
| 9 | The end. | 39 |

(5) **General Remarks :**

(a) Even though the subject matter of the *Sūtras* was the same, certain rules (Geometry) which are described by *Baudhāyana* are not included in the text by the other three writers, even though they are mathematically more correct.

(b) **It is a surprise to find that all 4 Śulba Sūtras. omit certain constructions which are Geometrically important and are used in the construction of citis.**

**These are :—**

(1) Each diagonal of a rectangle bisects it.

(2) The diagonals of a rectangle or a square, bisect one another and divide the figure into four parts. (a) In the case of a rectangle, the two and two opposite parts (triangles) are similar and equal to one another. (b) The four triangles so formed are all equal in area.

(3) In the case of a square, the diagonals divide the square into four parts, all of which are similar and equal to one another but also they are equal in area,

(4) This construction is used in moulding bricks.

For further description see the article on bricks.

(5) The circle can be divided into any number of equal parts, such as 8. 16, 32 and 64 by drawing the diameters.

(Used in the construction of *Paricāyya* and *Rathacakra citis*).

(6) To inscribe a square as large as possible in a circle by joining the ends of diameters, situated on the circumference of a circle. The diameters are at rt. angles to one another. This is used in the construction of *Gārhapatya* and a number of other *citis.*

(7) A triangle can be divided into a number of equal and similar parts by dividing the sides into an equal number of parts and then joining the points of division. Used in the construction of *Prauga* and *Ubhayataḥ Prauga Citis*: by moulding bricks of that shape.

(8) There are a few more constructions also which are not described here.

Important items as quoted below, require a thorough scrutiny before they are accepted as correct, these are :-

1) The measure of an *Aṅgula.*

2) From where the geometrical rules and constructions used in this work were received.

3) What the *Śulba Sūtras* have to say regarding the visit of Py thagoras to India.

4) The shapes and sizes of bricks used in the construction of various *citis*.

5) Measures of *Prakarma* and *Puruṣa* and their use.

6) The altars of all the *citis*. whether different in shape, were constructed having the same area.

7) The shapes of different *citis* vary according to the school of ***Yajurvda***.

**Conclusions :**

1) This book is mainly written, with a view to make the general public realise our progress and position in the case of Geometry in those days.

(a) . India laid the foundation of the three branches of Mathematics namely (1) Arithmetic, (2) Algebra and (3) Geometry.

(b) As the period of the development of Geometry is prior to the era of Pythagoras, the question of Greek influence in this case does not arise.

2) I would conclude this Introduction with a request to all the teachers and professors to imbibe on the minds of youngsters – that as the foundation was laid by us, it is also our duty to see that the edifice is also properly built by us.

(3) There is no dearth of Mathematicians in India to achieve this, provided they are made to understand their duty in this respect.

(4) With the advance of the binary System, the use of the decimal system (9 numerals and 0) will come to an end. One cannot stop this advance but effort should be made by contributions from our side to achieve the same.

(5) I would like that all the Indian authors writing books on Mathematics make a mention of the achievements of Indian Mathematicians with their names and offer their respects to their predecessors in this respect.

(6) On account of want of proper guidance and ignorance of the subject proper, I might have committed mistakes. I request the reader to bring them to my notice so as to enable me to correct them in my writings in future.

I hope this work will meet with the approval of scholars.

**S. D. Khadilkar**

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## ABBREVIATIONS

Āp. Sl. = Āpastamba Śulba.
Āp. Sr. = Āpastamba Śrauta.
Baudh. Sl. = Baudhāyana Śulba.
Baudh, Śr. = Baudhāyana Śrauta.
Kāt. Śl. = Kātyāyana Śulba.
Kāt. Śr. = Kātyāyana Śrauta,
Ś. Br. = Śātapatha Brāhmaṇa.
T. S. = Taittirīya Saṁhitā.

(1) *Āpastamba Śulba Sūtra* Published and printed by the Mysore University is used in compiling this book.

(2) *Reference to the Sūtras and Adhyāyas* : Published by Dr. Thibauṭ in the magazine *Pandit* is made in this case.

(3) *Kātyāyana Śulba Sūtra* : Published by *Acyuta Granthamālā* is used, as a reference in this case.

# KĀTYĀYANA ŚULBA SŪTRA

*With*

## Translation and Notes

## *CHAPTER I*

रज्जुसमासं वक्ष्यामः । का. शु. १.१.

"We shall explain the additions or subtractions to be made by means of a cord" (in measuring the sacrificial areas). Kāt. Śl. 1.1.

Ācārya Kātyāyana who has described the various pandals such as *prāgvaṁśa*, *sadas* and *havirdhāna* etc., with detailed dimensions, has said that the *mahāvedi* is 36 *prakramas* in length in the East-West direction, the West side is 30 *prakramas* and the east 24 *prakramas*. He has also said that the area of the altar on the first sacrifice should be 7½ sq. *puruṣas* and this area should be increased by 1 sq. *puruṣa*, at the time of each subsequent or additional sacrifice. The alter at the time of the first sacrifice is called *saptavidha*. The altar at the last sacrifice is called *ekaśatavidha* which has an area of 101½ sq. *puruṣas*.

All this is possible only when the East-West line is properly demarcated. There is likely to be a slight variation, on account of personal error, which should be avoided. With a cord, the East-West line can be properly located. The sun is the sole guide in this respect. The sun rises in the East and sets in the West, and these points will not be determined without the help of a cord.

In order to avoid all these mishaps and get a correct picture of the different altars like the *sautrāmaṇikī vedi*, the *paitṛkī vedi*, the *mahāvedi* and the two-fold (द्विगुण) and three fold (त्रिगुण). *vedis* of *Aśvamedha*, the author of the Kātyāyana Śulba-sūtra has taken upon himself the task of introducing the complicated rules and methods of constructing those altars with accurate measurements.

Śulba-sūtra is a manual which contains aphorisms (*sūtras*) laying down the rules for adding, deducting and measuring areas by the use of a cord. In the two rules 'समचतुरस्राणामुक्तः समासः' (Kāt. Śl. 2.21) and 'नाना प्रमाणसमासे' (Kāt.Śl. 2.22) the method of summation or of addition is explained; while in the rules 'चतुरस्राच्चतुरस्रं निर्जिहीर्षन्' (Kāt. Śl. 3.1) and 'समचतुरस्रं दीर्घचतुरस्रम्' (Kāt. Śl. 3.4) the method of deduction is described. Thus this *Sūtra*-text deals with the rules for measuring altars.

समे शङ्कुं निखाय शङ्कुसम्मितया रज्ज्वा मण्डलं परिलिख्य यत्र लेखयोः शङ्क्वग्रच्छाया निपतति तत्र शङ्कू निहन्ति सा प्राची । का.शु. १.२.

"Having fixed a pole, on a level piece of ground (which is to be measured) and having described a circle by a cord attached to the pole, one should mark the points with pegs, where the shadow of the top of the pole touches (due to the rising sun) the circle, the line joining these pegs is the West to East line." Kāt. Śl. 1.2.

This would have been true if the sun would have travelled from Due-East to Due-West. But this is not so. The sun is inclined towards the North from 21st December to 21st June called *Udagayana*. During this period, the sun will be closer to *Citrā*. The sun is inclined to South or *Svātī* from 21st June to 21st December. This period is called *Dakṣiṇāyana*. But on the days of equinoxes, that is when the sun is exactly on the Equator or when the sun is passing through the centre of both *Citrā* and *Svātī*; or on the 21st March and 21st September, when the sun is at *Meṣa* or *Tulā* it is exactly Due-East.

But as this is not possible for the day-to-day working, the East fixed as per rules already stated, prior to the commencement of the sacrifice, should be taken as Due-East till the end of the sacrifice.

तदन्तरं रज्ज्वाऽभ्यस्य पाशौ कृत्वा शङ्क्वोः पाशौ प्रतिमुच्य दक्षिणाऽऽयम्य मध्ये शङ्कुमेवमुत्तरतः सोदीची । का.शु. १.३.

"Having increased the length of the cord, (which is equal to the distance of the two pegs fixed in the East-West line), by itself, one should provide two slings at each end of the cord. Fix the slings on the two pegs (already fixed) stretch the cord with its mid-point towards the South and fix a peg at the place, where the mid-point of the cord touches the ground. He should similarly proceed towards the North. The line joining these pegs will be the South to North line" Kāt. Śl. 1-3.

रज्ज्वन्तयोः पाशौ करोति । का.शु. १.४.

"One should provide slings at both ends of the cord". (Kāt. Śl. 1.4). Provision of a sling at each end of the cord is necessary for correct measurements.

श्रोण्यंसनिरञ्छनसंख्यासमासभङ्गेषु लक्षणानि । का. शु. १.५.

EF = Measuring cord

Points A and B = *Aṁsas*

Points C and D = *Śroṇis*

Point H = *Saṁkhyā-samāsabhaṅga*

Point G = *Nirañchana*

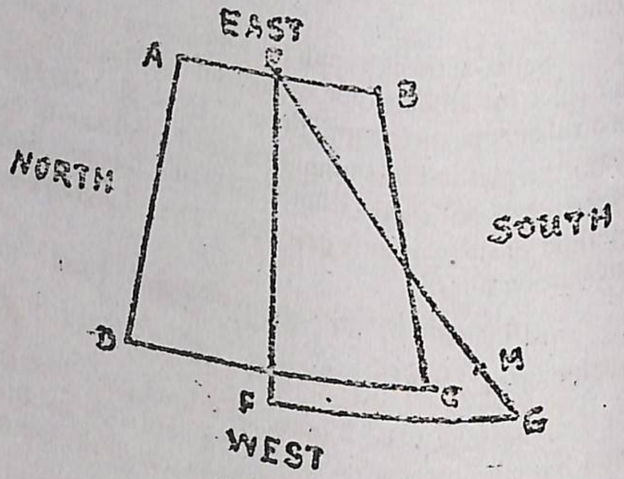

Fig 1

"One should make a mark on the cord, at the spots which are meant to fix the places of the two *śroṇis*; the two *aṁsas*; the *nirañchana* and at the meeting point of the two cords called *saṁkhyāsamāsabhaṅga*." Kāt. Śl. 1-5.

In this case, EF = The measuring cord = the extended cord = East-West line. The two North-East and South-East corners are called *aṁsas*, the two North-West and South West-corners are the *śroṇīs*. *Nirañchana* is a mark on the extended cord. This is done in two ways, In one case, it is a mark on the fourth part of the extended cord and in other case, it is on the sixth part of the extended cord. In both these cases, the mark is to be made just near the joint of both the extended cord and the measuring cord. This joint is called *saṁkhyāsamāsabhaṅga*.

This mark of *nirañchana* helps in setting out right angles, with the help of the rt. ∠ed △s, the units of lengths of whose sides are 3,4 and 5; and 5, 12 and 13 respectively.

The marks of *śroṇī* and *aṁsa* are dependent on the measurements of the *vedi*, on the East and West sides. These measurements will depend on the shape of the *vedi*. The two opposite parallel sides of a trapezium are unequal. In a triangle, there is a base but no opposite side. Thus the measurements on the East side for the *aṁsa* will not be the same as on the West side for the *śroṇī* in the case of a trapezium, but in the case of a triangle, there will be no measurements for the *aṁsa*.

At times, the whole area near the East side is considered as *aṁsa* and the whole area along the West side is considered as *śroṇī*.

In the above sketch, the extended cord is equal to the measuring cord, the *nirañchana* mark G is at $\frac{1}{4}$th of the extended cord and nearer to the meeting point H of the two cords. Slings are provided at both ends E and F. The *nirañchana* mark is stretched towards South with the result that a rt. ∠ed △ EFG is formed.

By stretching this *nirañchana* mark either on the South or on the North, one can measure the North-South lengths on the East or on the West and mark the *śroṇīs* or the *aṁsas* of an altar as desired.

Thus before starting any construction, all the above marks must be made on the measuring cord to be used.

प्राच्यन्तयोः शङ्कू निहन्ति । का.शु. १.६.

" One should fix a peg at each end of the East-West line (the distance between the two pegs must be equal to the measuring cord)". Kāt. Śl. 1.6.

These pegs will be of great assistance in measuring areas.

श्रोण्योरंसयोश्च । का. शु. १.७.

" One should fix pegs at the points of both the *śroṇīs* and *aṁsas*". Kāt.Śl.1,7.

शङ्कोः पाशौ प्रतिमुच्य निरञ्छनेन गृहीत्वा दक्षिणपूर्वां दिशं हरन्ति । का.शु.१.८.

"Having fixed the slings, on both the pegs of the East-West line, one should stretch the cord (which is already doubled) by the nirañchan.. mark to the South-East and fix a peg (at the point where the *nirañchana* mark meets the ground)." Kāṭ. Śl. 1.8.

एवमुत्तरतः । का.शु. १.९.

"The same procedure should be followed towards North". Kāt. Śl. 1.9.

विपर्यस्येतरतः । का.शु. १.१०.

"Having exchanged the slings, one should reverse the process on the other side." (i.e. on the West). Kāt. Śl. 1.10.

In *sūtra* 6 above, the author asks the sacrificer to fix pegs on the East-West line at a measured distance apart. Then pegs are fixed at the points of *śroṇīs* and *aṁsas*. How this is to be achieved by stretching the *nirañchana* mark is explained in *sūtras* 7-10. All these pegs are necessary for demarcating the area which is to be measured or with their help, *vedis* of different shapes are to be constructed.

स समाधिः सर्वत्र । का. शु. १.११.

"This method of construction is to be followed in all cases". Kāt. Śl. 1.11.

प्रमाणमभ्यस्याभ्यासचतुर्थे लक्षणं करोति तन्निरञ्छनम् । का. शु. १.१२.

"Having increased the length of the measuring cord, by its own length, one should make a mark on the fourth part of the increased cord (which is nearer to the joint of the original measuring cord and the increased cord). This mark is called the *nirañchana*". Kāt. Śl. 1.12.

अक्ष्णया तिर्यङ्मानीशेषः । का. शु. १.१३.

"When the North-South length of the cord *Tiryaṅmānī* (= तिर्यङ्मानी) is subtracted from the total length of the cord (original length of the measuring cord + an equal length of the extended cord), what remains is the hypotenuse." Kāṭ. Śl. 1-13.

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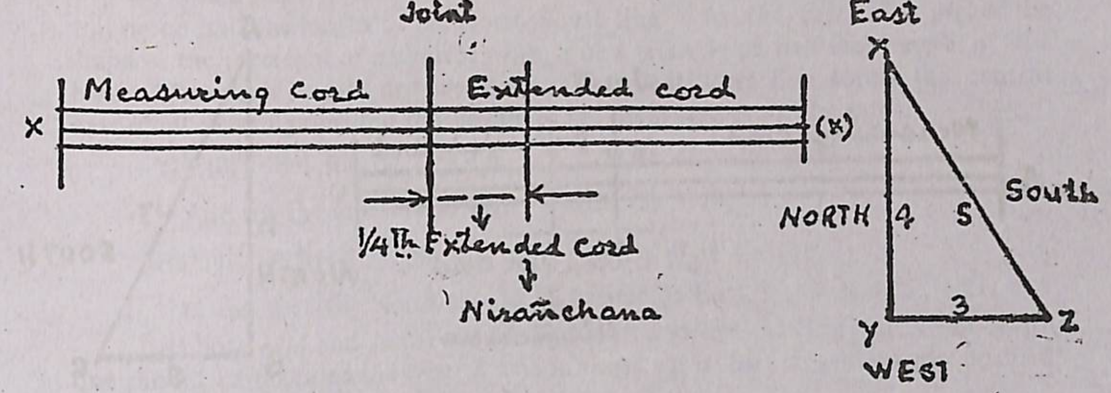

Fig 2

Measuring Cord = *Pramāṇa Rajju* = *Daṇḍa Rajju* ( प्रमाणरज्जु = दण्डरज्जु ).

Now : measuring cord. = XY = extended cord = 4 units.

∴ measuring cord + extended cord = 4 + 4 = 8 units.

¼th of the extended cord = 1.

North – South side = YZ = 3.

Hypotenuse = XZ = 5.

∴ (Measuring cord + Extended cord) – North South side.

= hypotenuse = XZ = 8 – 3 = 5.

From the above sketch, it is obvious that the *nirañchana* mark will produce only one rt. ∠ed △, the lengths of the sides being of 3, 4 and 5 units.

प्रमाणार्धं वाऽभ्यस्याभ्यासषष्ठे लक्षणं करोति तन्निरञ्छनम् । का. शु. १.१४.

"Having extended the measuring cord by its half, one should make a mark on the sixth part of the extended (half) cord. (This mark should be nearer the joint of the original and extended (half) cord). This is the *nirañchana* mark". Kāt. Śl 1-14.

अक्ष्णया तिर्यङ्मानीशेषः । का.शु. १.१५.

"When the North-South side is subtracted from the total length of the measuring cord, what remains is the hypotenuse". Kāt. Śl. 1.15.

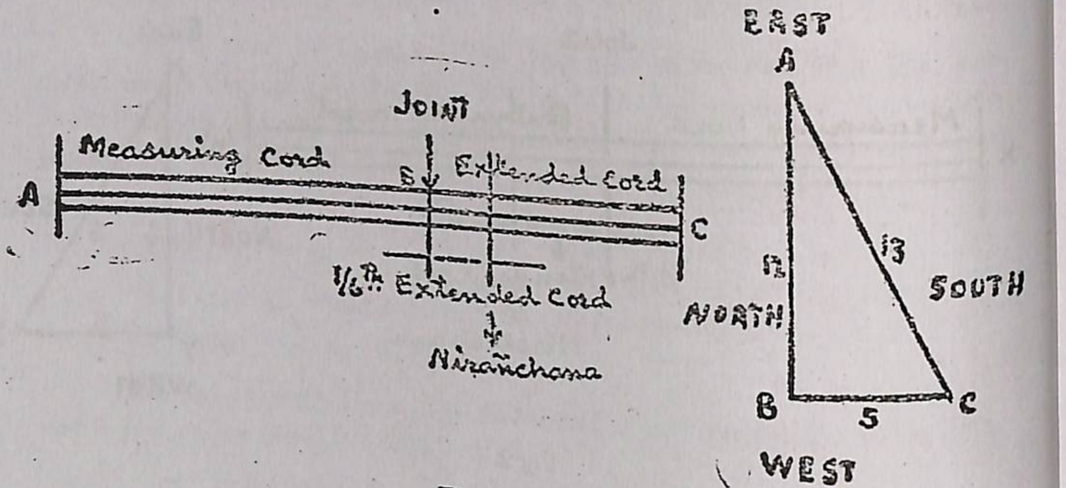

Fig 3

AB = measuring cord = 12.

BC = extended cord = ½ measuring cord = 6.

Measuring cord + Extended cord = AB + BC = 12 + 6 = 18.

18 – North South side = 18 – 5 = 13 = Hypotenuse.

This method of constructing a rt.∠ed △ is only useful, when the lengths of the sides are of 5, 12 and 13 units or its multiples.

Both these rules (Kāt. Śl. 1-12 and 14) do not lead us to any general rule for the formation of a rt. ∠ed △.

प्रमाणार्धे समचतुरस्रस्य शङ्कुः । का. शु. १.१६.

"One should drive a peg, at half the length of the measuring cord, at the time of fixing the corners of the square". Kāt. Śl. 1.16.

शास्त्रवदर्धे दीर्घचतुरस्रस्य । का. शु. १.१७.

"In the case of an oblong, one should drive a peg at half the measure of the North-South line, as prescribed in the holy text." Kāt. Śl. 1.17.

शकटमुखस्य चैवम् । का. शु. १.१८.

"One should follow the same procedure (as described above) in the case of a *citi*, the shape of which is just like the forepart of a chariot." Kāt. Śl. 1.18.

एतेन प्राग्वंशवेदिमानानि व्याख्यातानि । का. शु. १.१९

"Hereby the measurements (construction) of the *prāgvaṁśa. havirdhāna*, and such other halls along with different altars, should be considered as explained." Kāt. Śl. 1.19.

In the case of the rules from 16 to 19, the author wants to impress upon the reader, that one should provide marks for fixing *śroṇis* and *aṁsas*, as per measurements already prescribed. In the case of a square, the mark will be on half the length of the measuring cord. In the case of a rectangle, the mark will be on half the length of the North-South line. In the case of a *citi* of the shape of the forefront of a chariot *prauga* or a triangle on half the length of the base. These half lengths are necessary as the East-West line forms the central axis of all these figures and the distances are measured from the same.

शाळामानं च । का. शु. १.२०.

"And for the measure of *śālā*". Kāt. Śl. 1.20.

तत्रोदीची प्राचीवत् । का. शु. १.२१.

"In this case the North should be treated as East". Kāt. Śl. 1.21.

As both *śālā* and *sadas* are facing North and not the East, the North-South line should be taken as the central axis in those cases for measurements instead of the East-West line.

सदसश्चैवम् । का. शु. १.२२.

"And similarly for the *sadas*." Kāt. Śl. 1.22.

अपरिमितं प्रमाणाद्भूयः । का. शु. १.२३.

"The word immeasurable (*aparimita*) when used in the context of measurement means that the measure is to exceed than the already stated definite measurement". Kāt Śl. 1·23.

Whenever one comes across the word immeasurable it should be taken to mean that it is slightly in excess over what has already been stated.

As regards the *vedi* it is said '*aparimitā vā*' (Kāt Śr. 5-3.14), and in the case of *cayana* it is described as '*aparimiteṣṭakā*' (Kāt. Sr. 17-7-29). In both these cases '*aparimita*' does not mean 'immeasurable' but the measurement is slightly in excess.

प्रमाणे शास्त्रं प्रमाणं निर्ह्रासविवृद्ध्योः । का. शु. १.२४.

"In the case of any increase or decrease in the measure, one should follow the unit of measure prescribed by the Śulba." Kāt. Sl. 1.24.

योगश्च । का. शु. १.२५

"And also reasoning". Kāt. Śl. 1.25.

The increase or decrease should only be achieved with proper and reasonable steps as prescribed by the Śulba.

इतरस्य वितृतीये दक्षिणत इत्येतद्वक्ष्यामः । का. शु. १.२६.

"We shall now explain the rule : '*itarasya vitṛitiye dakṣiṇataḥ*' as quoted in" (Kāt.Sr. 4.8.18). or Kāt. Śl. 1.26.

In the place, where the sacrificer, offers oblations to the fire (1) the place of the *Gārhapatya*, should be located on the West and should be circular in shape. (2) from the centre of the place of *Gārhapatya*, the place of *Āhavanīya* is 11 to 12 *prakramas* on the East. The shape of the *Āhavanīya* fireplace is a square. Having fixed the places of *Gārhapatya* and *Āhavanīya*, the place of *Dakṣiṇāgni* will be at a distance of *Vitṛtīya*, which will be explained now.

गार्हपत्याहवनीययोरन्तरालं षड्ढा सप्तधा वाऽऽगन्तुसमं त्रेधा विभज्यापरवितृतीय-लक्षणेन दक्षिणाऽऽयम्य तस्मिन्नग्निः । का. शु. १.२७.

"One should divide the distance between the centres of the *Gārhapatya* and *Āhavanīya* into 6 or seven parts and then add the sixth part or the seventh part to the six or seven parts already made. Then divide the whole distance into three parts; leaving aside one part from the western end, one should make a mark which is called *vitṛtīya*. (Fixing the slings on the pegs fixed at the centres of both the fires on the East and the West) stretch the cord with the *vitṛtīya* mark on the South, the place where the mark touches the ground is the centre of the place where the Dakṣiṇāgni is to be provided ". Kāt. Śl. 1.27.

The distance between the centres of the two sacred fires is 8, 11 or 12 *prakramas*. Divide this distance into six or seven parts. Add one part ($\frac{1}{6}$th to 6 parts or $\frac{1}{7}$th to 7 parts). Take a measuring cord of this length, provided with slings at both ends. Divide this cord into three equal parts. Make a mark on the third part which is nearer to the Western end. Fix slings on the pegs already driven at the centres of the two sacred fires on the East and the West Then with the mark (which is called *vitṛtīya*) in hand stretch the string towards South. The point where the mark touches the ground is the centre of the place of *Dakṣiṇāgni*.

विपर्यस्योत्तरत उत्करः । का. शु. १.२८.

"After having exchanged the slings, stretch the *vitṛtīya* mark towards the North. The point where the mark touches the ground will be the place for *utkara*". Kāt. Śl. 1.28.

This place is intended for throwing the earth and grass removed from the surface, while clearing and levelling the *vedi*. This place is on the North side and near the *cātvāla* but outside the *vedi*.

अपि वाऽन्तरत्रिभागोनया रज्ज्वा पूर्वार्धे समचतुरस्रं कृत्वा श्रोण्यामग्निः । का.शु.१.२९.

"Or else, after having made a square, by a cord, which is reduced by a third of the distance between the *Gārhapatya* and the *Āhavanīya* and which lies in the exterior half on the East side of the *Gārhapatya*, one should place the *Dakṣiṇāgni* in the centre of the South-West corner of this square". Kāt. Śl. 1.29.

विपर्यस्योत्तरांस उत्करः । का.शु. १.३०.

"After having exchanged the slings and having constructed a square in the exterior of the western half as above, one should fix the *utkara*, at the northern *aṁsa* of this square". Kāt. Śl. 1-30.

## CHAPTER II

*Note* :—The first chapter dealt with the important definitions from the Sulba-sūtra. It is termed as *Paribhāṣa*.

From the second chapter, of the Kātyāyana Śulba Sūtra, a start is made with the mathematical portion required for the various types of altars to be constructed. The following procedure will be adopted in dealing with this mathematical (Geometrical) portion.

**(1) The Sanskrit text from Kātyāyana will be quoted first. This will be followed by quotations of the same rule from Baudhāyana and Āpastamba.**

**(2) The English translation of the sūtra from Kātyāyana will be given.**

**(3) The rule will be proved Geometrically.**

**(4) This will be followed by comments. An attempt will be made to show why the rule was framed and lastly,**

**(5) The procedure followed by the sūtrakāras to prove the rule they have stated will be given .**

In the first five sūtras, Kātyāyana quotes a few measurements.

अङ्गुलै रथसंमितायाः प्रमाणम् । का.शु. २.१.

"The measure of the *vedi*, which is measured according to the dimensions of a chariot is the fingerbreadth (*aṅgula*) " Kāt. Śl. 2.1.

'रथमात्र्युत्तरा वेदिर्भवति' Kāt. Śr. Sū. 5.3.11. So says the Vedic text. The northern *vedi* is of the size of the front part of a chariot, and as all the dimensions of a chariot are measured in *aṅgulas* by one who prepares it, the *Vedi* is constructed by applying *aṅgulas* as the unit of measurement.

At the auspicious time of the *varuṇa praghāsa*, a *parvan* of the *Cāturmāsya* (in the sacrifice carried out during the four months) the northern *vedi* must have the size of the front part of a chariot and hence measurements are taken in *aṅgulas*.

The measurements quoted below are of the yoke of a chariot.

तत्राष्टाशीतिशतमीषा । का.शु. २.२.

"The pole is 188 *aṅgulas* in length". Kāt. Śl. 2.2.

The pole denoted the East-West length of a chariot (formed by the wooden pole).

चतुःशतमक्षः । का. शु. २.३.

"The axis is 104 *aṅgulas*." Kāt. Śl. 2.3.

This is denoted by the North South line of a chariot. (In other words this place is covered by *śroṇīs* of the northern altar).

षडशीतिर्युगम् । का. शु. २.४.

"The Yoke is 86 *aṅgulas*". Kāt. Śl. 2.4.

This is a piece of wood placed on the shoulders of the horses. The position of the yoke represents the breadth of the Vedi on the eastern side.

चत्वारोऽष्टकाः शम्या । का. शु. २.५.

"The yoke pin is 32 *aṅgulas*." Kāt. Śl. 2.5.

The yoke pin is a wooden stick inserted in the holes of a yoke, while tying the horses to a chariot. (This is also used in measuring the *cātvāla* pit). *Cātvāla* is a pit, the earth out of which is used for filling up the northern *vedi*.

पैतृक्यां द्विपुरुषं समचतुरस्रं कृत्वा करणीमध्ये शङ्कवः स समाधिः । का.शु. २.६.

[अ] यजमानमात्री चतुःस्रक्तिर्भवति इति एकेषाम् । बौ.शु. १.८३.

[ब] यजमानमात्री चतुःस्रक्तिर्भवतीति पैतृक्या वेदेर्विज्ञायते । आप.शु.पृ.९५.

[क] अथ दक्षिणेनान्वाहार्यपचनम् । चतुःस्रक्तिं वेदिं करोत्यवान्तरदिशोऽनुस्रक्तीः करोति चतस्रो वाऽवान्तरदिशोऽवान्तरदिशो वै पितरस्तस्मादवान्तरदिशोऽनुस्रक्तीः करोति । शत. ब्रा. २.६.१.१०.

" In the case of a *paitṛkī vedi* (to be constructed in the *Sākamedhaparvan* of the *Cāturmāsya* for the *Mahāpitryajña*) one should construct a square,having an area of 2 sq. *puruṣas*. Drive pegs at the mid points of the four sides. The lines joining these pegs form a square (having an area of 1 sq. *puruṣa*). This is the mode of construction". Kāt. Śl. 2–6.

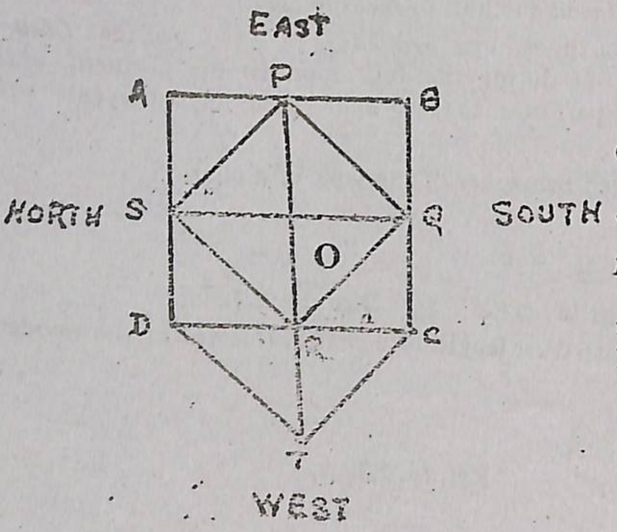

Fig 4

CT = DT = 1 *puruṣa*.

Let ABCD be a square, whose area is 2 square *puruṣas*.

P, Q, R and S are the mid points of the sides AB, BC, CD and DA respectively.

Join the points P, Q, R and S then the figures PQRS will be a square having an area of 1 sq. *puruṣa*.

Join PR and SQ, dividing the square ABCD into 4 small squares.

□APOS; □OPBQ; □OQCR and □ORDS

Each small □ is subdivided into two isosceles rt. $\angle$ed $\triangle$s by the sides of the square PQRS. These isosceles rt. $\angle$ed $\triangle$s are equal to one another.

$\therefore$ □PQRS = $\frac{1}{2}$ □ABCD = $\frac{1}{2} \times 2$ sq. *puruṣas*.

= 1 □ *purusa*.

In the case of *pitṛyajña* (Kāt. Śr. Sū) says 'दिक्स्रक्ति पुरुषमात्रं मिमीते' meaning thereby that a square whose corners are turned towards the cardinal directions will have an area of one sq. *puruṣa*.

NOTE :—(1) In this case one must know as to how to construct a square whose area is 2 sq. *puruṣas*.

(2) Then only he will be able to construct a square whose area is exactly half of the first square i.e. 1 sq. *puruṣa*.

Now this figure is an important indication as to how this must have led to **the discovery of the proposition that the diagonal of a square produces an area of double its size or in other words the square on the diagonal of a square is twice as large as that square.**

Let PR be the diagonal of a square PQRS and also the hypotenuse of the isosceles rt. $\angle$ed $\triangle$ PSR or $\triangle$ PQR.

$\therefore$ The square on PR = □ ABCD = □ PS + □ SR = 2 × □ PQRS.

Thus the diagonal PR of the square PQRS produces double the area of the square PQRS is proved.

**Thus it is not only the step leading to the wellknown proposition of the hypotenuse but is a clear proof of the same.**

करणी तत्करणी तिर्यङ्मानी पार्श्वमान्यक्ष्णया चेति रज्जवः । का.शु. २.७.

"*Karaṇī Tatkaraṇī Tiryaṅmānī Pārśvamānī* and *Akṣṇayā*; these are the five cords ". Kāt. Śl. 2-7 .

(a) *Karaṇī* is a cord, which is equal to the length of a rectilinear figure (or a cord by which the field is measured).

(b) *Tatkaraṇī* : A cord equal to a side, which produces so much i.e. a side of a square which will produce double the area of the existing square or three times the area. This side is termed as *Tatkaraṇī* or that-producer.

(c) *Tiryaṅmāṇī* : A cord equal to the length of the North-South side, which is always at rt. $\angle$s to the East-West side (*Prācī*). It may also be called as transverse side or short side or width.

(d) *Pārśvamānī* : The side which is parallel or not parallel to the East-West side. It is also called the longer side or length.

(e) *Akṣṇayā* : The diagonal of a square or an oblong. The hypotenuse of a rt. $\angle$ed $\triangle$.

पदं तिर्यङ्मानी त्रिपदा पार्श्वमानी तस्याक्ष्णया रज्जुर्दशकरणी। का.शु. २.८.

(अ) दशपदोत्तरा वेदिर्भवतीति सोमे विज्ञायते। आप.शु.पृ.९८; बौ.शु.१.९८.

(ब) दशपदामुत्तरवेदिं करोति। सत्याषाढ श्रौ.सू. ७.४.

"In a right $\angle$ed $\triangle$, whose side is one *pada*, and the other side is three *padas*, its hypotenuse will produce an area of ten sq. *padas*." Kāt. Śl. 2.8.

In the *Varuṇapraghāsaparvan* of *Cāturmāsya* sacrifice, the northern altar is a square having an area of ten square *padas*. 'दशपदोत्तरा वेदिर्भवति' says the text and hence the above rule is necessary.

एवं द्विपदा तिर्यङ्मानी षट्पदा पार्श्वमानी तस्याक्ष्णया रज्जुश्चत्वारिंशत्करणी। का.शु.२.९.

(अ) उत्तरवेदिं निवपति चत्वारिंशत्पदां युगमात्रीं वा....। का. श्रौ.सू. १७.३.१४.

(ब) तां वै युगमात्रीं वा सर्वतः करोति, यजमानस्य वा दशदश पदानि।

शत.ब्रा.२.५.१.३४.

(यजमानस्य वा दशदश पदानीत्यत्र सर्वतः करोत्यस्यानुवृत्तौ—दशदशपदा शतफलक्षेत्रफलका इत्यर्थो भवति....शुल्बपरिशिष्टकारस्तु दशपदक्षेत्रफलकामाह।)

"In a right $\angle$ed $\triangle$ whose one side is two *padas* and the other side six *padas*, the hypotenuse will (be a *tatkaraṇī* of a square) produce an area of 40 sq. *padas*". Kāt. Śl, 2.9.

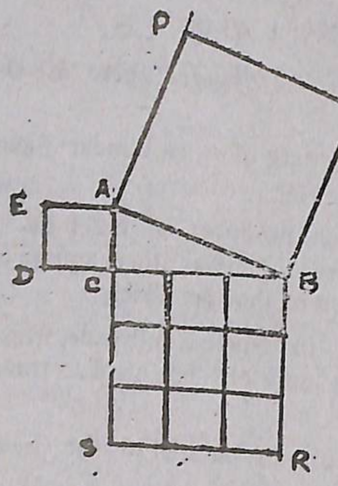

Fig 5

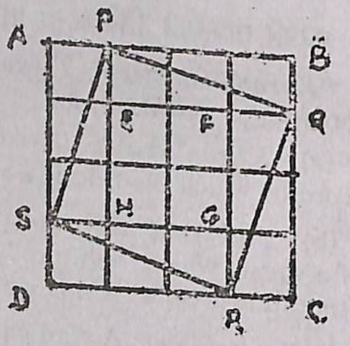

Fig 6

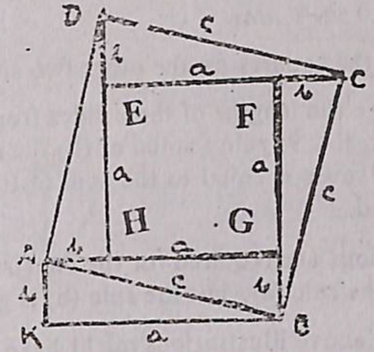

Fig 7

(a) Fig. 5 : ABC is a rt. $\angle$ed $\triangle$.

The side $AC = 1$ *pada* and $BC = 3$ *padas*.

$\therefore$ The hypotenuse AB will produce an area equal to 10 sq. *padas*.

This is quite clear from the figure.

Side AB of the square $= \sqrt{10}$ *padas*.

(b) Fig. 6 : There are in all 16 small squares in the $\square$ ABCD.

Each small $\square$ has an area of 1 sq. *pada*.

Now $\square PQRS = \triangle PQE + \triangle QRF + \triangle SRG + \triangle SPH + \square EFGH$

and each of the 4 triangles has an area $= \frac{1}{2} \cdot 3 \cdot 1 = \frac{3}{2}$ sq. *padas*.

and the $\square$ EFGH $= 4$ sq. *padas*.

$\therefore \quad \square PQRS = 4 \cdot \frac{3}{2} + 4 = 6 + 4 = 10$ sq. *padas*.

PQRS is a square produced on the diagonal of a rt. $\angle$ed $\triangle$PBQ whose side $PB = 3$ *padas* and $BQ = 1$ *pada*.

$\therefore$ **The conclusion is that the square produced on the diagonal is equal to the sum of the squares containing the right angle.**

(c) Fig. 7 : ABK is a rt. $\angle$ed $\triangle$.

Sides $AK = b$, $KB = a$ and $AB = c =$ hypotenuse.

Now ABCD is a square on AB = sq. on the hypotenuse $= C^2$,

$\therefore \quad C^2 = \triangle ABG + \triangle BFC + \triangle CED + \triangle HDA + \square EFGH.$

Area of each $\triangle = \frac{1}{2} a \cdot b$.

The area of the $\square$ EFGH $= (a-b)^2$

$\therefore \quad AB^2 = BK^2 + AK^2 \qquad \therefore \quad C^2 = 4 \cdot \frac{1}{2} a.b + (a-b)^2$

$2ab + a^2 - 2ab + b^2 = a^2 + b^2.$

By substituting $a = 3$ *padas* and $b = 1$ *pada*

The value of the square on the hypotenuse $= C^2 = a^2 + b^2 = 3^2 + 1^2 = 10$ sq. *padas*

Thus all the three figures prove the statement given above that the square on the hypotenuse = 10 sq. *padas*.

= The sum of the squares on the other two sides = 1 + 9 = 10 sq. *padas*.

Now if we change the lengths of these sides from 1 and 3 to 2 and 6 units. as are given in the (Kāt. Śl.2.9) rule quoted at (b) the same general rule regarding the square on the hypotenuse is equal to the sum of the squares on the other two sides is clearly illustrated.

All these constructions are required for the construction of *Uttaravedis*. All the *Śulbakāras* quote the rule (a), but the rule (b) is given by Kātyāyana only.

To assume that the above illustrations might have induced the authors of the *Śulbasūtras* to the discovery of the theorem of the hypotenuse is far from correct and one will have to see whether more illustrations of the type quoted above are available.

**These two are the instances where no rectangles or squares are mentioned but only triangles are used and the word *Akṣṇayā* meaning hypotenuse indicates that both are right ∠ed △s.**

उपदिष्टं युगप्रमाणं शम्याप्रमाणं च दर्शनात् । का.शु. २.१०.

"The measures of *Yuga* and *Śamyā* are already explained because these are prescribed (in the scriptures)". Kāt. Śl. 2.10.

It is prescribed in the *Śatapatha Brāhmaṇa* (1.5) and (1.24) that the northern altar should be of the measure of *yuga* or a *śamyā*. Similarly Kātyāyana states (Kāt. Śr. Sū. 17.3.13) that the northern altar in the *Cayan*, should be constructed with the measure of *yuga* and again (in the Kāt. Śr. Sū. 5.3.29) the northern altar should be prepared with the measure of a *śamyā*, at the time of *Varuṇapraghāsa*

दीर्घचतुरस्रस्याक्ष्णया रज्जुस्तिर्यङ्मानी पार्श्वमानी च यत्पृथग्भूते कुरुतस्तदुभयं करोतीति क्षेत्रज्ञानम् । का.शु. २.११.

(अ) दीर्घस्याक्ष्णया रज्जुः पार्श्वमानी तिर्यङ्मानी च यत्पृथग्भूते कुरुतस्तदुभयं करोति । आप.शु.पृ. २१.

(ब) दीर्घचतुरस्रस्याक्ष्णया रज्जुः पार्श्वमानी तिर्यङ्मानी च यत्पृथग्भूते कुरुतस्तदुभयं करोति । बौ.शु. १.४८.

"In a rectangle, its diagonal produces by itself an area, which its length and breadth produce separately. This is the way of measuring the areas". Kāt:Śl:2-11.

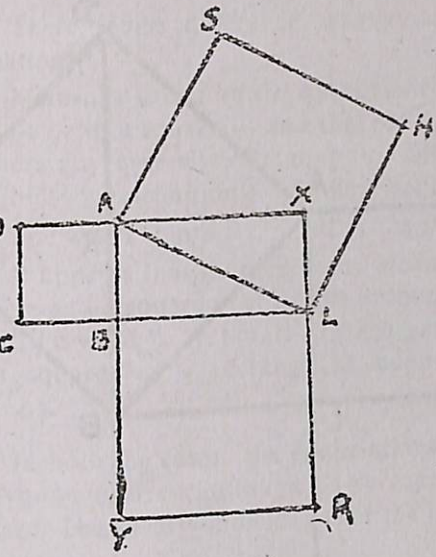

Fig 8

ABLX is a rectangle.

The sides AB and BL are unequal. All included ∠s are rt. angles,

AB = XL = East-West side = *Pārśvamānī*. AX = BL = North South side = *Tiryaṅmānī*.

AL = diagonal or hypotenuse = *Akṣṇayā*.

The rule states that the area produced by the hypotenuse AL=area produced by the side AB + area produced by the side BL.

**In other words this is the enunciation of the theorem of the hypotenuse which is known as the 'Pythageras Theorem'** which states : In the case of a rt. ∠ed △ the area produced by the hypotenuse is equal to the sum of the areas produced by the other two sides (which include a rt. angle).

The next rule that follows also states a similar general enunciation. But this enunciation pertains to a square.

समचतुरस्रस्याक्ष्णया रज्जुर्द्विकरणी । का.शु. २.१२.

[अ] समचतुरस्रस्याक्ष्णया रज्जुर्द्विस्तावतीं भूमिं करोति । बौ.शु. १.४५.

[ब] चतुरस्रस्याक्ष्णया रज्जुर्द्विस्तावतीं भूमिं करोति । आप.शु.पृ.२६.

"In a square its diagonal produces an area which is double that of its side." Kāt. Śl. 2.12.

All its sides are equal and angles are rt. ∠s.

AL = BK = East-West-line
= *Pārśvamānī*.
AK = BL = North-South-line
= *Tiryaṅmānī*.
AB = hypotenuse = *Akṣṇayā*.

The rule states that the area produced by the diagonal AB of a square is equal to twice the area produced by its side.

∴ □ ABCD = □ALRY + □LBSV

(This is according to the rule already stated above).

But as the sides of a square are equal to one another,

□ABCD = 2 × □ ALRY

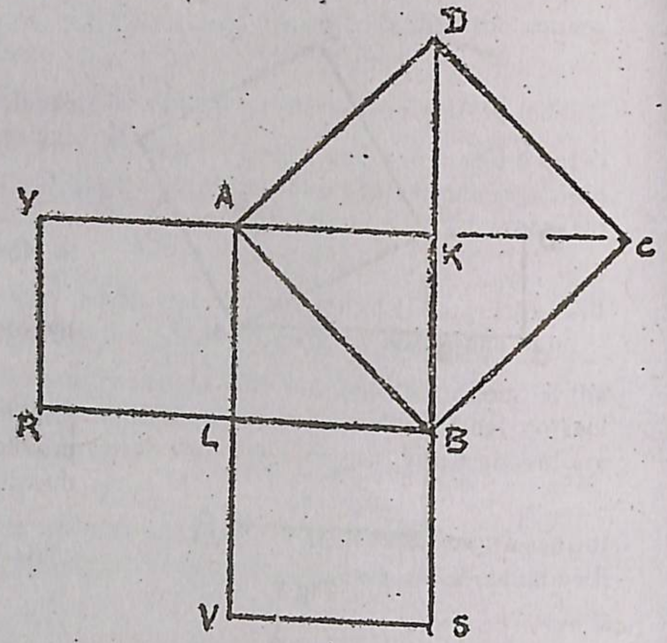

Fig 9

(A) ∴ $AB^2 = AL^2 + BL^2$, that is $AB = \sqrt{2}.BL$.

(B) The diagonal AB divides the square AKBL into two isosceles rt. ∠ed △ s. namely △ ABL and △ AKB.

Similarly the square ABCD on the diagonal AB is divided into 4 equal rt. ∠ed △ s ( isosceles) by the diagonal AC and BD. All these are equal to one another.

And □ ALRY and the □ LBSV are both divided into △ s which are equal to one another.

Since the △ s whether on the □ formed by the diagonal AB or the △ s formed on the squares of the sides of the original □ are all equal, we have :

□ ABCD = 4 x △ AKB = 2. △ ALR + 2. △ BLV
= □ AYRL + □ BLVS
= 2 □ AYRL

That is, the square on the diagonal AB of a square is twice the square on its side.

This is another version of the general theorem of the hypotenuse as already stated above. In the first case of a rectangle the sides were unequal. It was a general case. While in the case of a square the sides are equal.

The difference between the enunciation of the theorem of the hypotenuse as stated by the *Śulbakāras* and that of Pythagoras lies in the words : a rectangle, a square and a rt. ∠ed △.

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There is no difference whatsoever in the statements of all the three enunciations.

Naturally therefore the question arises as to why the *sūtrakāras* refer to a rectangle or to a square — and that too just at the start of each enunciation and was there any necessity of stating two different enunciations for the same purpose when only one definition would have served the purpose ?

The words " in a rt. ∠ed △" is the correct beginning that is desired.

It appears that there is some definite intention in using these words ( a rectangle and a square ) before the authors. It is clear that both the enunciations start with a word "a rectangle" (*dīrghacaturasra*) or "a square" (*samacaturasra*). Now a square or a rectangle is bound by four sides and included angles are rt. ∠s.

In both the cases, the enunciations refer only to two sides and the diagonal ( hypotenuse ), even though a reference at the start is made to a rectangle or to a square. This clearly indicates that the author wants to refer to a rt. ∠ed△ and its hypotenuse.

Now if the author really wanted to refer to a rectangle or to a square he would have referred to all the four sides in the enunciation itself and not to two only as is done here. Then both those enunciations would have been as follows :

" In the case of a rectangle the area produced by the diagonal will be equal to half of that produced by all the sides taken together separately[1]".

" The diagonal of a square produces an area which will be equal to half of that produced by all its sides taken together separately[2] ".

Really speaking the author wants to deal, with the case of a rt. ∠ed △and show the relation of the hypotenuse and the other two sides.

All the authors of the *Śulbasūtras*, while treating the portion of Geometry make a beginning with the propositions of squares and rectangles. Their first object is to find out the side of a square which is equal in area to the sum of two squares or the difference of two squares, or to transform a circle, a triangle, a rectangle or a polygon into a square. For all these constructions, the principle of the theorem of Pythagoras is applied to get the solution.

The area of the first basic *citi* was 7½ sq. *puruṣas*. Out of these a square of 4 sq. *puruṣas* formed the main body called *Ātman*, a certain portion formed the two wings and the remaining formed the tail. All these have a square shape.

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(1) दीर्घचतुरस्रस्याक्ष्णया रज्जुः पार्श्वमानी तिर्यङ्मानी च यत्पृथग्भूते कुरुतः तदर्धं करोति ।

(2) समचतुरस्रस्याक्ष्णया रज्जुः अर्धभूमिं करोति ।

Both the books of *Kātyāyana* and *Āpastamba* refer to these chapters as :
" This is the second chapter which deals with squares[1]". Kāt : Śl :
" The ways of constructing a square and its derivations[2] ". Āp : Śl :

In these chapters, the author mainly refers to squares and rectangles. And hence he has made a reference to the square and a rectangle in enunciating these propositions.

Really speaking there was no necessity of having two general enunciations at all. One would have best served the purpose. But in order to make the matters easy, two separate enunciations have been prepared : (1) One deals in rectangles, required for the construction of pandals, meeting places etc., (2) The other deals in squares. This was required for the construction of *Aśvamedha* altar or the semi-circular place for the *Dakṣiṇāgni*.

करणीं तृतीयेन वर्धयेत्तच्च स्वचतुर्थेनात्मचतुस्त्रिंशोनेन सविशेष इति विशेषः । का. शु. २.१३.

(अ) प्रमाणं तृतीयेन वर्धयेत्तच्च चतुर्थेनात्मचतुस्त्रिंशोनेन स विशेषः । बौ. शु. १.६१.

(ब) प्रमाणं तृतीयेन वर्धयेत्तच्चतुर्थेनात्मचतुस्त्रिंशोनेन सविशेषः । आप. शु. पृ. २६.

" One should increase the measure (The side of a given square whose double producer is to be found) by its third part and again by the fourth part of this third less by the the thirty-fourth part of this fourth. The value thus obtained is called the *saviśeṣa*." (This is the difference between a *karaṇī* and its *dvikaraṇī*). Kāt : Śl : 2.13.

This rule is discussed in detail, in a separate article namely " To find the value of $\sqrt{2}$ ".

प्रमाणं तिर्यग् द्विकरण्यायामः तस्याक्ष्णयारज्जुस्त्रिकरणी । का. शु. २.१४.
बौ. शु. १.४६ आप. शु. पृ. ३५.

" One should take the breadth equal to the measure of a square and length equal to the diagonal of that square, then the diagonal of such an oblong will produce an area equal to three times the area produced by the side of the square. This diagonal is called *trikaraṇī* " Kāt : Śl : 2.14.

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[1] चतुरस्रादिनिरूपिणी द्वितीया कण्डिका । का. शु. कं. २.

[2] चतुरस्रसाधनोपायः तदुपपत्तिश्च । आप. शु.

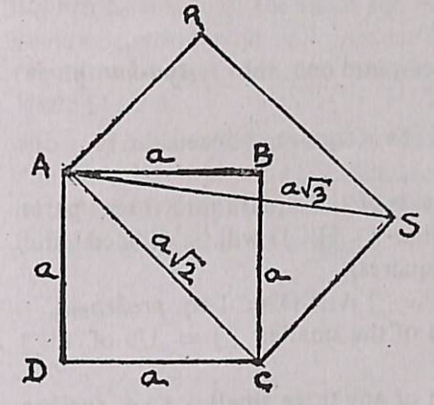

Fig 10

Let ABCD be a square,

where $AD = AR = a$.

$\therefore \; AC = a \times \sqrt{2}$.

Then ACSR will be an oblong or a rectangle.

$\therefore \; AS^2 = AC^2 + AR^2 = 2a^2 + a^2 = 3a^2$

That is the area produced by the diagonal of an oblong, the lengths of whose sides are $\sqrt{2} \cdot a$ and $a$ will be equal to three times the area produced by the side of a square (a).

(A) The *dvikaraṇī* or the two fold producer or the value of $\sqrt{2}$ was useful for :

(1) Constructing the *Aśvamedha Vedi* which is twice in area of the basic *Vedi*.

(2) For constructing the altar for *Dakṣiṇāgni* (semicircle) equal in area to that of a circle,

(3) Constructing a *Paitṛkī Vedi*-which is a square of two sq. *puruṣas* in area.

(B) The *trikaraṇī* is useful for constructing the *Aśvamedha Vedi*, which is three times in area of the basic *Vedi*, whose area is $7\frac{1}{2}$ sq.*puruṣas*.

तृतीयकरण्येतेन व्याख्याता । का. शु. २.१५. बौ. शु. १.४७. आप.शु. पृ.३६.

" Thereby the cord which will produce the third part of the area is also explained ". Kāt : Śl : 2.15.

प्रमाणविभागस्तु नवधा । का. शु. २.१६. बौ. शु. १.४७. आप.शु. पृ. ३६.

" When a *tṛtīya-karaṇī* is required, divide the square into 9 parts " Kāt : Śl : 2.16.

करणीतृतीयं नवभागः । का. शु. २.१७.

" The third part of a side produces the ninth part of a square ". Kāt : Śl : 2.17.

नव भागास्त्रयस्तृतीयकरणी । का. शु. २.१८.

" Three ninth parts, out of nine, when combined into a square, produce an area which is one third of that square ". Kāt. Śl . 2.18.

सौत्रामण्यां प्रक्रमार्था । का. शु. २.१९.

" In order to explain the measure of a *prakrama* in the *Sautrāmaṇī* sacrifice, the *tṛtiya-karaṇī* is explained ". Kāt. Śl. 2.19.

तृतीयकरणी समासार्था । का. शु. २.२०.

" In order to combine three parts (squares) into one the *tṛtīya-karaṇī* is explained ". Kā : Śl : 2.20.

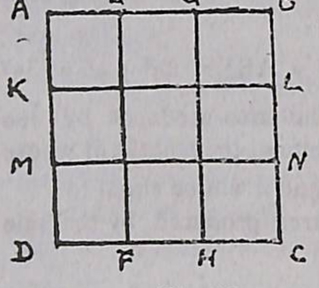

Fig 11

Let ABCD be a square, whose side is one *prakrama*.

Divide each side of the square into three parts and join them. Thus □ ABCD will be divided into nine equal parts. (squares).

The area of the □ ABCD = 1 sq. *prakrama*.

∴ The area of the smaller □ = 1/9 of a □ *prakrama*.

∴ The area of any three small □ s such as MNCD will be

$= CD \cdot MD = \frac{1}{3}\ pra. \times 1\ pra = \frac{1}{3}$ □ *prakrama*.

Thus (as per above rule), the side of a □ which will combine into one all the three above squares will be equal to a *prakrama*. This new measure of a *prakrama* is to be used for the construction of the *Sautrāmaṇī Vedi*, this *Vedi* is similar in shape to the *Mahāvedi* and will have an area which will be $\frac{1}{3}$rd of the *Mahāvedi*. How to combine these three small squares is already explained.

**2nd Method :** Let AD be a straight line. Divide it into three equal parts. On one side AB, construct a square AYRB.

Join AR. With radius AR and A as centre, cut the line AD at L. Join YL.

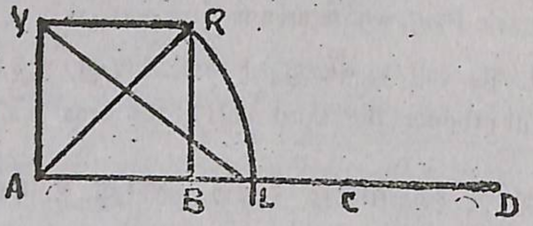

Fig 12

Now $YL^2 = AY^2 + AL^2$.

$\therefore\ AR = AL$

$\therefore\ AR^2 = AB^2 + BR^2$

$= AL^2$

$\therefore\ YL^2 = AY^2 + AB^2 + BR^2$

$= 3AB^2$ as $AB = 1/3\ AD$.

$\therefore\ AB^2 = 1/9\ AD^2$

$\therefore\ YL^2 = 3AB^2 = 3.\ 1/9\ AD^2 = 1/3\ AD^2$

∴ The □ on YL will have $\frac{1}{3}$rd area of the □ on AD.

तुल्यप्रमाणानां समचतुरस्राणामुक्तः समासः । का. शु. २.२१.

" How to combine two equal squares is already explained". Kāt : Śl : 2.21.

नानाप्रमाणसमासे ह्रसीयसः करण्या वर्षीयसोऽपच्छिन्द्यात्तस्याक्ष्णयारज्जुरुभे समस्यतीति समासः । का. शु. २.२२.

( अ ) नानाचतुरस्रे समस्यन् कनीयसः करण्या वर्षीयसो वृध्रमुल्लिखेत् वृध्रस्य अक्ष्णया रज्जुः समस्तयोः पार्श्वमानी भवति । बौ. शु. १.५०

( ब ) तुल्ययोश्चतुरस्रयोरुक्तः समासः । नानाप्रमाणयोश्चतुरस्रयोः समासः ह्रसीयसः करण्या वर्षीयसो वृध्रमुल्लिखेत् । वृध्रस्याक्ष्णयारज्जुरुभे समस्यति । तदुक्तम् । आप. शु. पृ. ३९

" To combine two unequal squares, one should cut off from the bigger square by a side of the small square a rectangular portion. The diagonal of this rectangle, so formed will produce an area equal to the area produced by both (small and large) squares. This is the method of combining unequal squares ". Kāt : Śl : 2.22.

How to combine equal squares is already explained by Kāt : Śl : 2.12. similarly Kāt : Śl : 2.11 enunciates the rule for combining two unequal squares but the rule Kāt : Śl : 2.22 not only states the same thing but the *śulbkāras* give a definite clue as to how they came to the conclusion or a proof of the same.

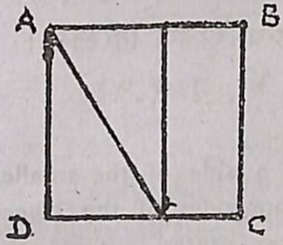

Fig 13

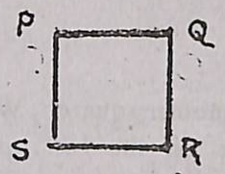

Fig 14

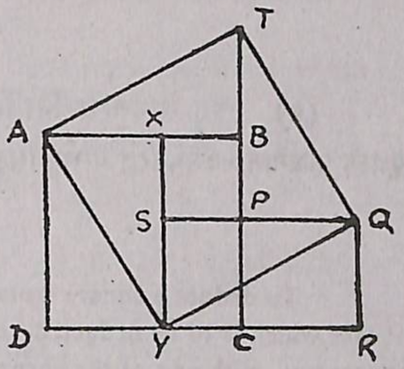

Fig 15

**Fig 13 and 14 :** Let ABCD be a large square and PQRS a small square. These two squares are to be combined into one square. Cut off sides AB and CD of the large square by a side PQ of the small □ PQRS at X and Y. Then the diagonal AY will produce a square equal in area of both the squares ABCD and PQRS. This is the hypotenuse AY of the rectangle AXYD which produces an area equal to the sum of the squares on sides AD and DY.

**2nd Solution : Fig. 15.**

It is proposed to combine the □ ABCD and □ PQRC.

From sides AB and CD cut off AX = DY = PQ = X.

∴ Prove that $AD^2 + DY^2 = AY^2$

i. e. □ ABCD + □ PQRC = □ATQY.

∵ △ ADY + △ AXY + △ SQY + △ YRQ + □ PSXB = □ATQY.

∴ △ ADY = △ ABT = △PQT = △ RQY = △ AXY = △ SQY.

∴ △AXY + △ ATB + △ PTQ + △ SQY + □ SPBX = □ ATQY.

$\therefore\ AD^2 + DY^2 = AY^2 = AD^2 + X^2$

*Note* :— (1) This is a solution obtained by division and appropriate readjustment.

(2) This is nothing but the same general theorem which is already explained. (Kāt : Śl : 2.11). It is repeated again in somewhat different words. In that rule it was simply enunciated but here the same is tried to be proved.

## CHAPTER III

चतुरस्राचतुरस्रं निर्जिहीर्षन् यावन्निर्जिहीर्षेत्तावदुभयतोऽपच्छिद्य शङ्कू निखाय पार्श्वमानीं कृत्वा पार्श्वमानीसंमितामक्ष्णयां तत्रोपसंहरति स समासेऽपच्छेदः सा करण्येव निर्ह्रासः। का. शु. ३.१.

(अ) चतुरश्राच्चतुरश्रं निर्जिहीर्षन् यावन्निर्जिहीर्षेत्तस्य करण्या वृध्रमुल्लिखेत्। वृध्रस्य पार्श्वमानीमक्ष्णयेतरत्पार्श्वमुपसंहरेत्। सा यत्र निपतेत्तदपच्छिन्द्यात्। छिन्नया निरस्तम्।
आप. शु. सू. ४१-४२.

(ब) चतुरस्राच्चतुरस्रं निर्जिहीर्षन् यावन्निर्जिहीर्षेत्तस्य करण्या वर्षीयसो वृध्रमुल्लिखेत् वृध्रस्य पार्श्वमानीमक्ष्णयेतरत् पार्श्वमुपसंहरेत् सा यत्र निपतेत्तदपच्छिन्द्याच्छिन्नया निरस्तम्।
बौ. शु. १.५१.

" To deduct a square from another square. With a side of the smaller square which is to be deducted, cut off the two opposite sides of the bigger square and with one of the corners of the new rectangle as centre, cut off the side of the bigger square, so that this radius (which is equal to the side of the bigger square) will be the hypotenuse of a rt. ∠ed △. The length of the unknown side will produce an area equal to the difference of two squares. " Kāt : Śl : 3.1.

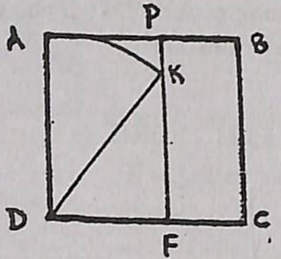

Fig 16

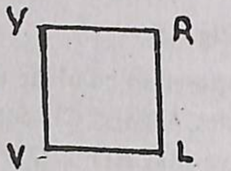

Fig 17

**1st Method :** To deduct the small square YRLV from the big square ABCD from AB and DC cut off AP and DF equal to YR (a side of □ YRLV ), Join PF with centre D and radius DA cut off PF at K. Join DK. The square on KF will be equal to the difference of two squares.

$$\therefore \quad DK^2 = KF^2 + DF^2 = AD^2$$

$$\therefore \quad KF^2 = DK^2 - DF^2 = AD^2 - DF^2.$$

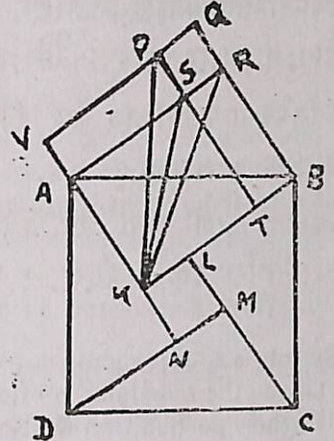

Fig 18

2nd Method : Fig. 2.

VQBK = big square.

ASTK = small square (to be deducted).

Cut off VP = KT = KA = BR = a side of the small square ASTK.

Then □ ABCD = □ KBQV + □ KAST

If KB = $a$ = a side of the big square,

and AS = $b$ = a side of the small square,

and hypotenuse = AB = C,

Then □ ABCD = △ AKB + △LBC + △ MCD + △ AND + □ KLMN.

In △ AKB; BT = KB - KT = $a - b$.

Since all [the 4 rt. ∠ed △s : △ AKB, △ BLC, △ CMD, △ DNA are similar and equal the side KL of the □ KLMN = $a - b$.

∴ □ KLMN = □ PQRS.

Now rect. ARBK + rect. KVPT + □ PQRS = △ AKB + △ ABR + △ PKT + △ VPK + □ PQRS.

= □ KBQV + □ ASTK = □ ABCD

∴ □ ABCD - □ ASTK = □ KBQV ∴ $a^2 = c^2 - b^2$.

The above two methods are the solutions of the rule quoted in the *śulba* which are entirely dependent on the thorough knowledge of the theorem of the hypotenuse.

*Āpastamba* states this rule followed by an illustration " The side drawn across becomes the diagonal (of a certain rectangle). Let it be the generator of the fourfold, (of a certain square). It produces (a square equal to) both the squares which are produced separately by the cut off portion (of the opposite side) and the other side. (i. e. the shorter side of the rectangular segment of the larger square or the side of the other square). If the other side be one *puruṣā* the remainder will be three sq. *puruṣas*. This has been stated before.[1] " *Āpa*. Śl : pages 43–44.

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(1) उपसंहृताऽक्ष्णयारज्जुः सा चतुष्करणी छिन्ना चेतरा च यत्पृथग्भूते कुरुतस्तदुभय करोति। तिर्यङ्मानी पुरुषं शेषस्त्रीन्। तदुक्तम्। आ.शु.सू. ४३-४४.

To transform a reactangle into a square.

दीर्घचतुरस्रं समचतुरस्रं चिकीर्षन्मध्ये तिर्यगपच्छिद्यान्यतरद्विभज्येतरत् पुरस्तात दक्षिणतश्चोपदध्यात् शेषमागन्तुना पूरयेत्तस्योक्तो निर्हासः। का.शु. ३.२.

(अ) दीर्घचतुरस्रं समचतुरस्रं चिकीर्षन् तिर्यङ्मान्याऽपच्छिद्य शेषं विभज्योभयत उपदध्यात्। खण्डमागन्तुना संपूरयेत्। तस्य निर्हासः उक्तः। आप.शु.पृ. ४६.

(ब) दीर्घचतुरस्रं समचतुरस्रं चिकीर्षंस्तिर्यङ्मानीं करणीं कृत्वा शेषं द्वेधा विभज्य विपर्यस्येतरच्चोपदध्यात् खण्डमावापेन तत् संपूरयेत्तस्य निर्हासः उक्तः। बौ.शु. १.५४.

" One desiring to transform a reactangle into a square should cut off (from the rectangle) a square with its shorter side. Divide the remaining portion of the rectangle into two equal parts. Add the farther portion, on the South side of the square on the East and complete the square by introducing a small square. The method of deducting a square is already explained ". Kāt : Śl : 3.2.

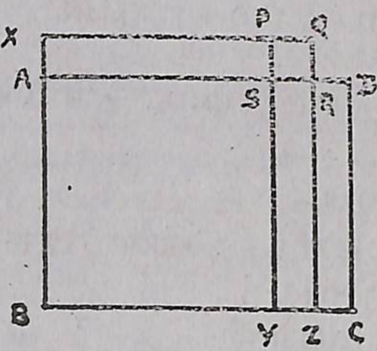

Fig 19

Let ABCD be a rectangle.

The longer side AD = $a$

shorter side AB = $b$

□ ABYS = $b^2$

SD = AD - AS = $a - b$.

$\therefore$ SR = $\frac{a-b}{2}$. RZ divides the remaining rectangle SYCD into two equal parts SYZR and RZCD.

Place reactangle RZCD at ASPX and complete the square BZQX.

$\therefore$ Rect. ABCD = □ XBZQ - □ PQRS

$$\therefore\ ab = (BY + YZ)^2 - SR^2$$

$$= \left(b + \frac{a-b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$$

$$= \left(\frac{a+b}{2}\right)^2 - \left[\frac{a-b}{2}\right]^2$$

$$\therefore\quad \left[\frac{a+b}{2}\right]^2 = \left[\frac{a-b}{2}\right]^2 + ab.$$

By eliminating the extra square PQRS, a new square can be formed as desired. This is already explained in the above rule. (difference of two squares).

NOTE—Any number of rt. ∠ed △s can be obtained from the above formula by substituting $a = n^2$ and $b = 1$.

अतिदीर्घं चेत्तिर्यङ्मान्याऽपच्छिद्यापच्छिद्यैकसमासेन समस्य शेषं यथायोगमुपसंहरेदित्येकः समासः। का.शु. ३.३.

" If the rectangle is too long, one should cut the longer side again and again by the breadth and combine the squares into one square. The remaining portion can then be adjusted after reasonable adjustments. This is the way as to how a rectangle is transformed into a square. ". Kāt : Śl : 3.3.

To transform a square into a rectangle.

समचतुरस्रं दीर्घचतुरस्रं चिकीर्षन् मध्येऽक्ष्णयाऽपच्छिद्य तच्च विभज्यान्यतरत् पुरस्तात् उत्तरतश्चोपदध्याद्विषमं चेद्यथायोगमुपसंहरेदिति व्यासः। का.शु. ३.४.

(अ) समचतुरस्रं दीर्घचतुरस्रं चिकीर्षन् यावच्चिकीर्षेत् तावतीं पार्श्वमानीं कृत्वा यदधिकं स्याद्यथायोगमुपदध्यात्। आप.शु.पृ.४७.

(ब) समचतुरस्रं दीर्घचतुरस्रं चिकीर्षंस्तदक्ष्णयाऽपच्छिद्य भागं द्वेधा विभज्य पार्श्वयोरुपदध्याद्यथायोगम्। बौ.शु. १.५२.

" One who desires to transform a square into a rectangle, should divide the square into two parts by its diagonal. Divide one part of this again into two. Place one of these parts on the East and other on the West. If the rectangle is too long (i. e. The longer side is not the double of the smaller) one should add these portions suitably so as to fit the two sides of the other half." Kāt : Śl : 3.4.

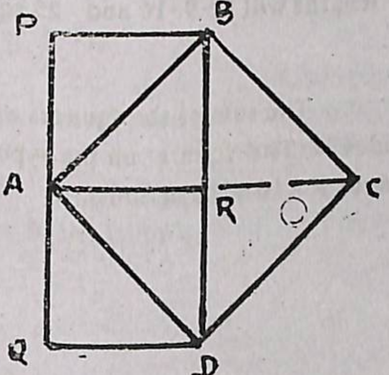

Fig 20

Let ABCD be a square.

Divide this square by its diagonal BD into two parts ABD and BCD.

Divide BCD again into two parts by RC (half of the diagonal AC) such as △ BRC and △ RCD.

Place △ BRC as △ PAB and △ RCD as △ AQD. This will complete the rectangle PBDQ.

This construction is very important in as much as it gives a clear solution of the theorem of the square of the diagonal of a square.

Let ABCD be the square. Divide the same by its diagonals AC and BD into 4 rt. ∠ed △s ABR; BRC; CRD and ARD. Place △ BRC and △ CRD as APB and AQD.

So that □ ABCD = □ APBR + □ ARDQ.

∴ □ on AB = □ on PB + □ on QD = twice □ on PB.
AB is the diagonal of the □ APBR and PB its side.

∴ The square on the diagonal of a square is equal to twice the square on its side is proved.

प्रमाणं चतुरस्रमादेशादन्यत् । का.शु. ३.५.

"Unless otherwise specified the measure should be regarded as of a square." Kāt : Śl : 3.5.

द्विःप्रमाणा चतुःकरणी त्रिःप्रमाणा नवकरणी चतुःप्रमाणा षोडशकरणी । का.शु. ३.६.

"If a side (of the given square) is double the measure its area will be fourfold; thrice the measure will be ninefold; four times the measure will be sixteenfold.". Kāt : Śl : 3.6.

Thus the area produced by a side will be equal to a square on that side.

यावत्प्रमाणा रज्जुर्भवति तावन्तस्तावन्तो वर्गा भवन्ति तान्समस्येत् । का.शु. ३.७.

(अ) यावत्प्रमाणा रज्जुस्तावतस्तावतो वर्गान् करोति । आप.शु. पृ. ५७.

"As many units of a measure as are in a cord, so many squares of those units in that measure there will be in a square on that cord as a side, one should add them." Kāt : Śl : 3.7.

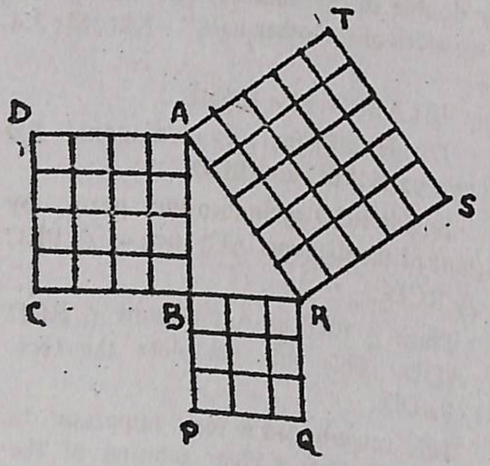

Fig 21

Let ABR be a rt. ∠ed △ having sides of lengths 3, 4 and 5 units. The number of units in squares of those lengths will be 9, 16 and 25 sq. units.

∴ The sum of the squares on the sides = The square on the hypotenuse or 9 + 16 = 25 sq. units.

अर्धप्रमाणेन पादप्रमाणं विधीयते । का.शु. ३.८.

"If the side of a given square is half the length of the given measure, the area produced by it (square) will be ¼th of the area (produced by the whole length of the measure)". Kāt Śl : 3.8.

तृतीयेन नवमोंऽशः । का.शु. ३.९.

"The area produced by 1/3 rd of the length of the measure will be 1/9th of the total area produced by that measure." Kāt. Śl. 3.9.

चतुर्थेन षोडशी कला । का.शु. ३.१०.

"The area produced by 1/4th of the length of a measure is 1/16th of the total area produced by the length of that measure". Kāt. Śl. 3.10.

If the measure is a fraction of the original measure, the area which is the square of that measure, decreases.

एष निर्ह्रासस्तस्य पुरस्तादुक्तं शास्त्रम् । का.शु. ३.११.

"The process of eliminating a small square from a larger one (or finding a square equal to the difference of two squares) is already explained. (Kāt : Śl : 3.1) above." Kāt : Sl. 3.11.

यावत्प्रमाणा रज्जुर्भवतीति विवृद्धिः ह्रासो भवति । का. शु. ३.१२.

"The area is either diminished or increased according to the measure of the cord." (see Kāt. Śl. 3.6 and 3.8–10) Kāt : Sl : 3.12.

चतुरस्रं मण्डलं चिकीर्षन्मध्यादंसे निपात्य पार्श्वतः परिलिख्य यदतिरिक्तं भवति तस्य तृतीयेन सह मण्डलं परिलिखेत् स समाधिः । का.शु. ३.१३.

"One desireous of circling a square, should draw half its diagonal from the centre towards the *aṁsa* and describe a part of the circle, towards a side with half the side of the square together with 1/3rd portion of the line which is in excess of the one, joining the centre and the *aṁsa* and half side of the square. Then this circle will be equal in area with that of the square." Kāt. Śl. 3,13.

मण्डलं चतुरस्रं चिकीर्षन् विष्कम्भं पञ्चदशभागान् कृत्वा द्वावुद्धरेच्छेषः करणी ।
का.शु.३.१४.

"One desireous of squaring a circle, should divide the diameter of the circle into 15 parts. Remove two parts. The remaining portion will form the gross value of a side of the equivalent square". Kāt. Śl. 3.14.

NOTE : For a general discussion and comments on the last two rules please refer to the following article. 'Squaring the Circle and Circling the Square.'

## CHAPTER IV

द्रोणचित रथचक्रचित् कङ्कचित् प्रउगचिदुभयतःप्रउगः समुह्यपुरीप इत्यग्नयः। का.शु. ४.१.

" *Droṇa*; *Rathacakra*; *Kaṅka*; *Prauga*; *Ubhayataḥ-Prauga*; *Samuhya-Purīṣa*, these are the types of *citis*." Kāt. Śl. 4.1.

In this way, after explaining the square and the circle, *Kātyāyana* gives a list of *citis*, which are six in number, *Suparṇa citi*, which consists of squares is fully described by *Kātyāyana* in the 16th chapter of his *Śrauta sūtra*. These *citis* are nothing but altars, constructed of bricks in various forms to receive the sacred fire. These *citis* are also called *Cayana*, *Agni*, *Agnicayana* etc. All these *citis* are described in the *Śatapatha-Brāhmaṇa*.

*Droṇa citi* is square in shape. The *Rathacakra* is circular. *Kaṅka* is a particular bird and the *citi* which resembles that bird is called the *Kaṅka citi*. The *Prauga* has the shape of the front pole of a chariot or a cart. Two triangles joined at the base make up the *Ubhayataḥ-Prauga*. A heap of earth, brought from outside and stored at one place is called *Samuhya-Purīṣa*. These are six different forms of *citis*.

द्रोणे यावानग्निः सपक्षपुच्छविशेषस्तावच्चतुरस्रं कृत्वा द्रोणदशमविभागो वृन्तमित्येके । का.शु. ४.२.

" In the case of the *Droṇa citi* one should construct a square of the total area, including that of *Ātman*, wings and tail. A tenth part of this whole area should be used as a stem. This is the view of some teachers." Kāt : Śl : 4.2.

तद्दशमेनापच्छिद्यापच्छिद्यैकसमासेन समस्य निर्हृत्य सर्वमग्निं तथाऽऽकृतिं कृत्वा पुरस्तात् पश्चाद्वोपदध्यात् । का. शु. ४.३.

" One should divide the whole area, into ten equal parts, remove one part and transform the remaining nine parts into one square. Turn the part that was left over into a square and attach the same either to the East or to the West (as a stem)" Kāt : Śl : 4.3.

मण्डलेऽप्येवम् का. शु. ४.४.

" In the case of a circular *citi*, the same procedure (of dividing the area into ten parts, and after removing one part, turn the remaining nine parts into a square and the square into a circle. Attach the one part, already left over after turning the same into a circle, to the circle already produced, either on the East or to the West as a stem) is to be followed ". Kāt : Śl : 4-4.

प्रउगे यावानग्निः सपक्षपुच्छविशेषस्तावद्द्विगुणं चतुरस्रं कृत्वा यः पुरस्तात् करणीमध्ये शङ्कुर्यौ च श्रोण्योः सोऽग्निः । का. शु. ४.५.

" In the case of a *Prauga citi*, (a triangular altar) one should construct a square of double the area, which is prescribed including that of wings and tail. A peg in the centre of the eastern side and two pegs on the *śroṇīs* will form a *Prauga* " Kāt : Śl : 4.5.

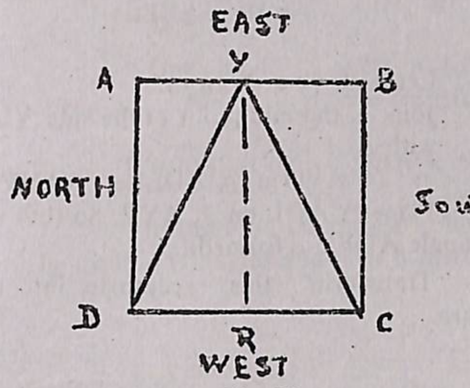

Fig 22

Let ABCD be a square. The area of □ ABCD = twice the area of △DYC. Fix a peg at Y the mid point of AB. Similarly fix pegs at C and D. The two points of *śroṇīs*.

Join DY and CY, thus △ CYD will be the *Prauga* required.

Mark the centre of CD at R. Join YR.

Place the △ YRC on △ AYD.

Thus the rectangle ADRY.

= rect. BCRY

∴ △ CYD = $\frac{1}{2}$ sq. □ ABCD.

उभयतःप्रउगे तावदेव दीर्घचतुरस्रं कृत्वा करणीमध्येषु शङ्कवः स समाधिः । का. शु. ४.६.

" In the case of an *Ubhayataḥ Prauga*- *citi*, one should first prepare a rectangle having double the area of the area of the *Ubhayataḥ Prauga citi* to be constructed. Then he should fix pegs at the mid-points of the rectangle and join them. This is the method of consruction of the *Ubhayataḥ Prauga* altar. " Kāt : Śl : 4.6.

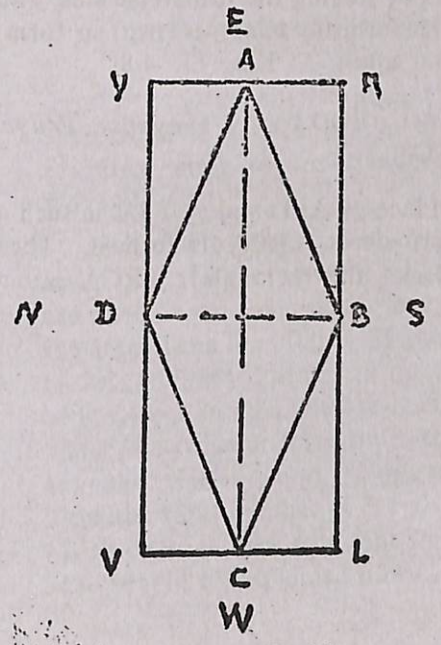

Fig 23

Let YRLV be a rectangle, whose area is double the area of the *Ubhayataḥ Prauga* to be constructed. Join the mid-points A, B, C, D of the sides of the rectangle YRLV. This will form two *Praugas* (triangels) ABD and BCD having a common base BD. The figure ABCD is called *Ubhayataḥ Prauga* and this as shown above is $\frac{1}{2}$ rectangle YRLV.

प्रउगं चतुरस्रं चिकीर्षन् मध्ये प्राञ्चमपच्छिद्य विपर्यस्येतरत उपधाय दीर्घचतुरस्रसमासेन समस्येत् स समाधिः । का. शु. ४.७.

" One who is desirous of transforming a triangle (isosceles) into a square, should divide the triangle into two parts. He should then place the triangles in such a way that a rectangle is formed. Then this rectangle should be transformed into a square. This is the method of transforming a triangle into a square. " Kāt : Sl : 4–7.

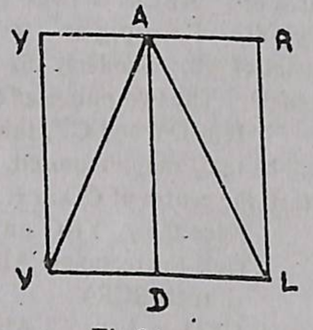

Fig 24

Let AVL be *a Prauga.*
Join D, the mid point of the side VL with A.
∴ △ AVD = △ ADL = △AYV.
Place △'ADL on △ AYV, So that a rectangle ADVY is formed.

Transform this rectangle into a square.

उभयतःप्रउगं चेन्मध्ये तिर्यगपच्छिद्य पूर्ववत् समस्येत् । का. शु. ४.८,

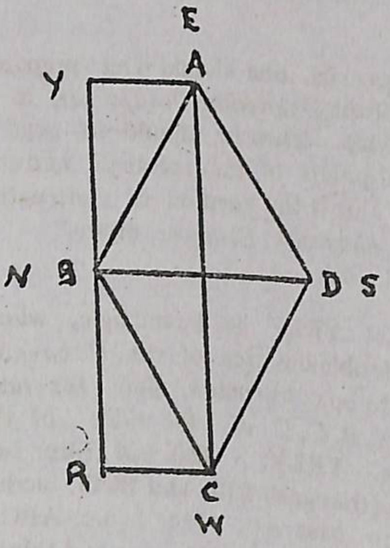

Fig 25

" In the case of the *Ubhayataḥ Prauga*, one should divide figure into triangles by joining the transverse side and then transform the triangles (two) so formed into a squre. " Kāt. Śl. 4·8.

Let ABCD be a *Ubhayataḥ Pruga.* Join AC.

Place △ ALD and △ DLC in such a way that a rect. YRCA is formed. Then transform this rectangle YRCA into a square.

एतेनैव त्रिकर्णसमासो व्याख्यातः । का. शु. ४.९.

" Hereby the transformation of a *trikarṇa* into a square should be regarded as explained. " Kāt : Śl : 4·9.

पञ्चकर्णानां च । का. शु. ४.१०.

" Also of a *Pañcakarṇa.* " Kāt : Śl : 4·10.

प्रउगेऽपच्छिद्यैककर्णानाम् । का. शु. ४.११.

" One should convert a figure having equal angles into a number of *Prauga* (triangles) and then transform them into a square ". Kāt. Śl. 4,11.

द्विकर्णानां समचतुरस्रेऽपच्छिद्य । का. शु. ४.१२.

" In the case of a figure having unequal angles, one should divide the figure into triangles and then transform them into a square. " Kāt. Śl. 4 12.

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## CHAPTER V

उत्तरेषु पुरुषोच्चयेनैकशतविधादित्येतद्वक्ष्यामः । का. शु. ५.१.

" Now we shall explain the statement contained in the *sūtra*. The increase of a square *puruṣa* in every successive fire altar starting from the sevenfold altar to the hundred and one fold altar. " (Kāt : Śr. sū 16·8·27). Kāt : Śl. 5.1.

The first *Agni*, constructed at the time of the first sacrifice was called seven-fold, the second eight-fold and the last one,hundred and one-fold.In this case the word 'seven-fold' means–one having an area of 7½ sq. *puruṣas*; if it is eightfold, the area of the fire altar is 8½ sq. *puruṣas*. Thus in the case of 101 fold *Agni* (*eakaśatavidha*) the area of the fire-altar will be 101½ sq. *puruṣas*. Thus there will be 95 *Agnis* in all. Add to these the three *Aśvamedha* fire altars namely those which are twice or three times the area of the first sevenfold fire altar. These are called *dvīguṇa* and *triguṇa* and the third one is called *ekaviṁśati–vidha.* Thus the total number of fire-altars to be constructed for these various sacrifices, which differ in area from one another will be 98. How this increase in area was achived will be explained now.

The increase in area of every fire altar should be followed by a corresponding increase in the area of bricks. The number of bricks should not be increased.

आद्योऽग्निर्द्विगुणस्त्रिगुणो भवतीति सर्वसमासः । का. शु. ५.२.

"The basic *citi* should be doubled in area, tribled in area." (Here the entire area of the basic *citi* should be taken into account. This rule is also stated by Kāt. Śr. Sū. 20.4.15). Kāt. Śl. 5.2.

The above *sūtra* applies to *Aśvamedha* sacrifice. The area of the altar to be constructed is either twice or thrice the area of the altar of the first basic *citi*. (This is $7\frac{1}{2}$ sq. *puruṣas*). This can be achieved by the introduction of *dvikaraṇī* or *trikaraṇī*. Thus the area of the *Aśvamedha citi* will be either 15 sq. *puruṣas* or $22\frac{1}{2}$ sq. *puruṣas*.

First construct a square of the total area ($7\frac{1}{2}$ sq. *puruṣas*), of the basic *citi*. The value of the diagonal of the above square will be equal to 38 *padas* + 9 *aṅgulas* less 2 *yavas* (i. e. 465 *aṅgulas* less 2 *yavas*). This diagonal will be the side of a square, which will produce an area, which will be double in area of the basic *citi*. This is the area of the 1st *Aśvamedha* altar, the total area being 15 sq. *puruṣas*. In this case the hypotenuse is nothing but the *dvikarṇī* or the double producer of the side of the square of the basic *citi*. The side of this square is $27\frac{1}{3}$ *padas* + 5 *yavas*, which produces an area of $7\frac{1}{2}$ sp. *puruṣas*.

Then with a side of the basic *citi* and the hypotenuse or the side of the square of the first *Aśvamedha* sacrifice construct another rectangle, the diagonal of this new rectangle will be a *trikarṇī* or three times producer of the area of the basic *citi*. The length of this *trikarṇī* is 47 *padas* + 5 *aṅgulas* + 2 *yavas*. The square produced by this side will have a total area of $22\frac{1}{2}$ sq. *puruṣas*. This is the area required for the sacrificial altar of the second *Aśvamedha* sacrifice.

एकविंशतिविधो भवतीति पुरुषाभ्यासः । का. शु. ५.३.

"Or else the *citi* should have an area of 21 sq. *puruṣas*." Kāt : Śl : 5.3.

Here is a third type of the sacrificial altar for the *Aśvamedha* sacrifice, the area of which is only 21 sq. *puruṣas*. The methods to be followed in increasing the area of the basic *citi*.

पुरुषाभ्यासे यावानग्निः सपक्षपुच्छविशेषस्तावच्चतुरस्रं कृत्वा तस्मिन् पुरुषप्रमाणमवदध्यात् । का. शु. ५.४.

"One should transform the area to be increased including that of the wings and tail into a square and then increase the same by one sq. *puruṣa*". Kāt : Śl : 5-4.

**1st Method**

समस्तं पञ्चदशभागान् कृत्वा द्वावेकसमासेन समस्येत् स पुरुषः । का. शु. ५.५.

"One should divide the whole area into 15 parts. Combine two of the parts into a square. This will be the new unit of measure of one sq. *puruṣa*". Kāt : Śl : 5.5.

The total area of the first basic *citi* is $7\frac{1}{2}$ sq. *puruṣas*. Divide this area by 15 and take two of these parts.

Its area will be :

$$7\tfrac{1}{2}/15 = 15/2 \times 1/15 = \tfrac{1}{2}\ (\text{a sq. } \textit{puruṣa}).$$

$$\therefore\ 2 \times \tfrac{1}{2} = 1 \text{ sq. } \textit{puruṣa} \text{ or } 2/15\,(7\tfrac{1}{2} + 1) = 1 + 2/15 \qquad \ldots \quad (A)$$

Thus the area of the *citi* for the second sacrifice will be :

$$7\tfrac{1}{2} + 1 = 8\tfrac{1}{2} \text{ sq. } \textit{puruṣas}.$$

(३) पञ्चविभागेन बृहती तस्य दशमविभागेन पादमात्री भवति । का. शु. ५.६.

"The area of the *Bṛhatī* brick will be 24 × 24 *aṅgulas* or 1/25th area of a squre *puruṣa*. Similarly the size of the *Pādamātrī* brick will be 12 × 12 *aṅgulas* or 1/100th area of sq. *puruṣa*". Kāt : Sl : 5.6.

**2nd Method :**

पुरुषं वा पञ्चमेनोभयतोऽपच्छिद्य पञ्चविभागान् समस्य तृतीयं निर्हृत्य तस्मिन् पुरुषप्रमाणेऽवदध्यादित्यपरम् । का.शु. ५.७.

"One should divide, each of the two sides of a square having an area of one sq. *puruṣa* into five equal parts. Then he should combine five of these (25 parts) into a square. After deducting 1/3rd of this, add the remainder to one sq, *puruṣa*. (This will be the new unit of measure of a sq. *puruṣa* to be used in the enlarged construction of altars). This is the 2nd method of arriving at the measure of a *puruṣa*". Kāt : Śl : 5.7.

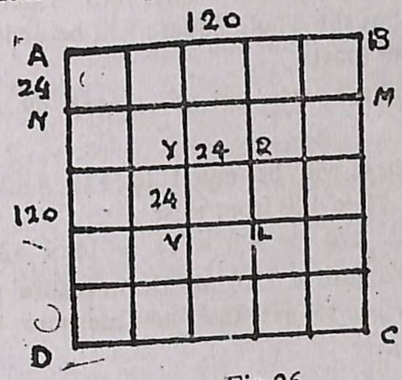

Fig 26

Let ABCD be a square 120 × 120 *aṅgulas* or (1 sq. *puruṣa*).

Divide the sides AB and BC into 5 equal parts. Thus the whole square will be divided into 25 parts. Take 5 of these parts (ABMN) 24 × 120.

Divide this into three part. Thus each part will be 8 × 120.

Deduct this part from the five parts.

Thus 24 × 120 – 8 × 120 = 16 × 120.

Add This (16 × 120) to a measure of 1 sq. *puruṣa* to get the measure o a *puruṣa* for the next sacrifice.

Thus $\frac{16 \times 120}{120 \times 120} = \frac{2}{15} \quad \because \quad 1 + \frac{2}{15}$ will be the new measure ... (B

पञ्चदशविभागोऽष्टाङ्गुलम् । का. शु. ५.८.

" The fifteenth part of a *puruṣa* is equal to 8 *aṅgulas* ". Kāt : Sl : 5·8.

In rule 5 above, one is advised to add two parts out of 15 to the sq. measure of a *puruṣa* i. e.

$2 \times 8 \times 120 = 16 \times 120$ sq. *aṅgulas*.

पञ्चारत्निर्दशवितस्तिर्विंशतिशताङ्गुलः पुरुष इत्येतस्माद् द्वादशाङ्गुलं पदमिति च ।

का. शु. ५.९

" A *puruṣa* consists of 5 *artnis* or 10 vitastis or 120 *aṅgulas*. This means that a *pada* consists of 12 *aṅgulas*. Kāt : Śl : 5·9.

This means that the 120th part of a *puruṣa* is equal to one *aṅgula*; and the fifth part (120/5 = 24 *aṅgulas* ) is equal to 24 *aṅgulas* = one *aratni*. And the 10th part, which is equal to 12 *aṅgulas* = one *pada* or one *vitasti*.

**3rd method :**

पुरुषं वा सप्तमेनोभयतोऽपच्छिद्य सप्तभागान् समस्य ससप्तमभागमङ्गुलं निर्हृत्य पुरुषप्रमाणेऽवदध्यादित्यपरम् । का. शु. ५.१०.

" Alternatively one should divide a square of one *puruṣa* into 7 parts both ways. Combine 7 parts out of these into a rectangle. Subtract from this (sum) a rectangle $1\frac{1}{7}$ *aṅgulas* by one *puruṣa* ($1\frac{1}{7} \times 120$) and then add the remainder to one sq. *puruṣa*. This is another method ". Kāt : Śl : 5·10.

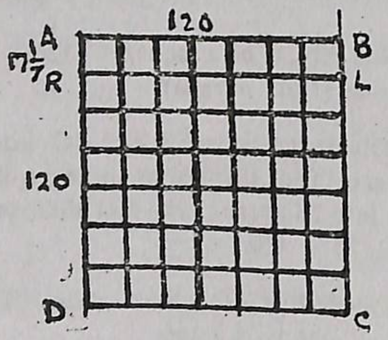

Fig 27

Let ABCD be a square 120 × 120 *aṅgulas*. Divide the sides into 7 parts each. Thus the whole square will be divided into 49 parts.

Take 7 parts out of these 49 as ABLR.

These will be equal to $17\frac{1}{7} \times 120$. Subtract $1\frac{1}{7} \times 120$ from this.

$\therefore \quad 17\frac{1}{7} \times 120 - 1\frac{1}{7} = 16 \times 120.$

Add this (16 × 120) to the measure of a sq. *puruṣa* to get the new measure of a *puruṣa*.

$$\text{Thus } 1 + \left(\frac{1}{7 \times 7} - \frac{1\frac{1}{7}}{120}\right) = 1 + \left(\frac{1}{7} - \frac{8}{7 \times 120}\right)$$

$$1 + \left(\frac{1}{7} - \frac{1}{7 \times 15}\right) = 1 + \frac{2}{15} \quad \cdots \qquad \text{(C)}$$

Thus the results obtained by all the three methods are the same. The ncrement of 2/15th of a sq. *puruṣa* to be added to one sq. *puruṣa* is the new measure of a *puruṣa* for the next sacrifice.

नारत्निवितस्तीनां समासो विद्यते संख्यायोगादिति श्रुतेः । का. शु. ५.११.

" *Aratni* and *vitasti* should not be increased, as their employment involves numbers in accordance with the *Śruti* " Kāt : Śl : 5.11.

*Aratnis* and *vitasti* should not be added to a *puruṣa*, as their increase is separately indicated in numbers, such as—

" चतुर्दशारत्नीन् दक्षिणे पक्ष उपदधाति चतुर्दशोत्तरे, चतुर्दश वितस्तीः पुच्छ उपदधाति' "

so says the Śruti, that " Add 14 *aratnis* to the southern wing, similarly add 14 *aratnis* to the northern wing and 14 *vitastis* to the tail. "

अथ त्रिपुरुषां रज्जुं मिमीते तां सप्तधा समस्यति तस्याश्चतुरो भागानात्मन्नुपदधाति त्रीन् पक्षपुच्छेषु । का. शु. ५.१२ ( अच्युत ग्रंथमाला )

" One should take a measuring cord, which can construct a square having an area of 3 sq. *puruṣas*. Divide this sq. into 7 parts. Use 4 parts (out of these seven) for the *Ātman* and use the remaining three for the wings and tail. " Kāt : Śl : 5.12.

( This rule is quoted in the text of *Acyuta Granthamālā* ).

With a measuring cord, construct a square having an area of 3 sq. *puruṣas*. Divide this square into 7 parts. 4 of these 7 parts should be added to *Ātman*, having an area of 56 sq. *puruṣas* in the 98th *Cayana*. And out of the three remaining parts, one part should be added to each of the two wings and a tail, having an area of 14 sq. *puruṣas*, excluding the numbers of *Aratnis* and *vitastis*.

In this way, in the case of 101-fold *Agni*, the *Ātman* will have an area of $57\frac{5}{7}$ sq. *puruṣas* and the two wings and the tail each $14\frac{3}{7}$ sq. *puruṣa*. The total area for the 101-fold *Agni* will be 101 sq. *puruṣa*. (57 5/7 + 3 × 14 3/7 = 57 5/7 + 43 2/7 = 101). And when the two wings are increased by an *aratni* each and the tail by *vitasti*. The total area so formed is 108 sq. *puruṣas*, and so *aratni* and *vitasti* are separately explained.

NOTE : This rule 12 quoted above is found in the *Śulba-sūtra* of *Kātyāyana* printed by the *Acyuta Granthamālā*..

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1. From commentary of *Mahīdhara*, P. 40 Acyuta Granthamālā,

## *CHAPTER* . VI

यथाग्नि वेदीष्टकाप्रमाणं वर्धत इत्येतद्वक्ष्यामः । का. शु. ६.१.

" The measurements of the altar and the size of bricks are increased ". Kāt. Śl. 6.1.

In the case of sacrifices, which follow one after the other in succession, the measure of a *puruṣa* is increased in every successive sacrifice. In order, therefore, to fill up completely the built up area of the fire–altar, with the same number of bricks, the size of the bricks has to be increased proportionately. As there is a distinct restraint, the shape and number of bricks to be used will not be changed on any account. Hence in order to keep the same shape and the same number, the size of the bricks is increased proportionately, as the size of the built up area of the altar.

या करणी चतुर्दशप्रक्रमान् संक्षिपति त्रींश्च प्रक्रमसप्तमभागान् स एकशतविधे प्रक्रमः। का. शु. ६.२.

" The side, which will produce a square, having an area of 14-3/7 *prakramas*; will form the measure of a *prakrama* in a *citi* of 101 fold fire altar ". Kāt. Śl. 6.2.

The side, which will produce a square, which will be equal in area of two squares, one of 14 *prakramas* and the other square of 3/7th of a *prakrama*, will be the measure of a *prakrama* in the last *cayana* of the 101 fold fire-altar.

Construct a rectangle, whose sides are 7 and 2 *prakramas*. Transform this rectangle, whose area is 14 sq. *prakramas* into a square.

Now taking the value of a *prakrama* = 3 *padas* = 36 *aṅgulas* its 3/7th will be = 36 × 3/7 = $15\frac{3}{7}$ *aṅgulas*. Therefore, the rectangle of the 3/7th of a *prakrama* = 36 × $15\frac{3}{7}$. Transform this into a square and add to a square of 14 *prakramas*. The side which unites these squares, will be the measure of a *prakrama* in the 101 fold *agni*. This *prakrama* is slightly less than 137 *aṅgulas*.

द्वितीये वा सप्तसु प्रक्रमेषु प्रक्रममवधाय तस्य सप्तमभागेन प्रक्रमार्थः । का.शु. ६.३.

" Or else, in the second fire–altar (or successive fire altars) one should add one more square *prakrama* to seven square *prakramas* and take the seventh part of this combination as a measure of a *prakrama* ". Kāt. Śl. 6.3.

प्रक्रमेण वा सप्तमभागेन प्रक्रमार्थः । का. शु. ६.४.

" One should increase the measure of a *prakrama* by its 1/7th and use the same as a measure of a *prakrama* ". Kāt. Śl. 6.4.

एवम् आ एकशतविधात् । का. शु. ६.५.

" In this way, upto an altar of 101 fold, one should take the measure of a *prakrama* increased by its 1/7th, at every successive fire-altar ". Kāt. Śl. 6.5.

नान्तःपात्यगार्हपत्ययोर्वृद्धिर्भवति तावदेव योनिर्भवति न वै जातं गर्भं योनिरनु वर्धत इति श्रुतेर्वृद्धेरत्यन्तं प्रतिषेधः । का. शु. ६.६.

" No increase is desired in the *Antaḥpātya* and *Gārhapatya* altars. This part is indeed the womb. There is a *Śruti*—the womb does not increase following the foetus which is born; so increase is prohibited ". Kāt. Śl. 6.6.

No increase is recommended in the size (area) of the *Antaḥpātya* and *Gārhapatya* fire-altar. The shape of the fire altar of *Gārhapatya* is circular. On the East of *Gārhapatya* or at the end of the *vedi*, at a distance of three *prakramas* the space for *Antaḥpātya* is described.

यावत्प्रमाणानि समचतुरस्राण्येकीकर्तुं चिकीर्षेदेकोनानि तानि भवन्ति तिर्यग्द्विगुणान्येकत एकाधिकानि त्र्यस्रिर्भवति तस्येषुस्तत्करोति । का. शु. ६.७.

" As many squares (of equal size) as one desires to combine into one, one should take the transverse side, equal to the measure of lengths of all squares (to be added) minus one and twice a side will be equal to the lengths of all the squares to which one more length of a square is added and a triangle formed. Then the perpendicular from the vertex to the mid-point of the opposite side of the triangle will sevre the purpose ". Kāt. Śl. 6.7.

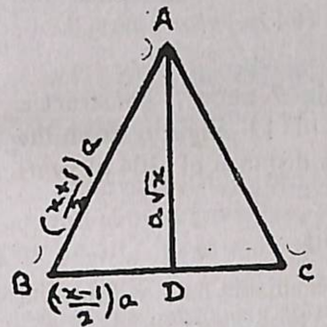

Fig 28

Let $a$ be the side of a square.

and $x$ = the number of squares to be added.

BC = base of the isosceles $\triangle ABC$ $(X-1)\cdot a$.

$$\therefore \quad BD = DC = \frac{X-1}{2}\cdot a$$

$$\text{and} \quad AB = AC = \frac{X+1}{2}\cdot a$$

$$\therefore \quad AB^2 = AD^2 + BD^2. \quad \therefore \; AD^2 = AB^2 - BD^2.$$

$$\therefore \quad AD^2 = \left(\frac{X+1}{2}\right)^2 \times a^2 - \left(\frac{X-1}{2}\right)^2 \times a^2$$

$$a^2 \cdot \left[\frac{X^2+2X+1-X^2+2X-1}{4}\right] = \frac{4X}{4}\cdot a^2 = a^2\cdot X$$

$$\therefore \quad AD = a\sqrt{X}$$

यथायूपं वेदिवर्धनमित्येतद्वक्ष्यामः । का. शु. ६.८.

" The altar should be increased in accordance with the number of sacrificial posts. This we shall explain ". Kāt. Śl. 6.8.

In the *Jyotiṣṭoma* sacrifice, one should tie either one or eleven animals to the sacrificial posts. This is being done in two ways:: (1) Some who follow the first method say that the eastern part, which is 24 *aratnis* long should be divided into eleven parts and the eleven posts should be fixed in those parts. (2) Those who follow the second method—advise to keep those 11 posts 104 *aṅgulas* apart.

In this way, the increase in the altar is explained. As this increase is effected by the measure of a *prakrama*, the same will be explained now. And:the increase in the *vedi* will only take place when 11 animals will be tied to the 11 posts, which will remain 104 *aṅgulas* apart.

या रज्जुरेकादशोपरवान् संक्षिपति दश च रथाक्षांस्तस्या यश्चतुर्विंशो भाग: स प्रक्रम: । का.शु. ६.९.

" The measuring cord, which is sufficient in length to measure the distance of 11 holes—created for erecting 11 posts—which are at a total distance apart (the last poles) equal to ten measurements of the axle (104 *aṅgulas*).—The 24th part of this measuring cord is a measure of a *prakrama* ." Kāt. Śl. 6.9.

The width of each pit is 12 *aṅgulas*. Therefore, the whole length of the cord will be – (10 × 104 + 11 × 12 = 1040 + 132 = 1172 *aṅgulas* i.e. 48 *aratnis*, 1 *pada* and 8 *aṅgulas* in length).

∴ The measure of a *prakrama* will be $\frac{1172}{24} = 48\frac{5}{6}$ *aṅgulas* i.e. slightly more than 2 *aratnis*.

तेन वेदिं निर्माय द्वादशाङ्गुलं पुरस्तादपच्छिद्य तद्यूपावट्याच्छङ्को: पुरस्तात् प्राञ्चमवधाय तस्मिन् यूपान् मिनोति । का. शु. ६.१०.

" One should measure the *vedi* with this cord, cut off 12 *aṅgulas* from the East of the *vedi* (altar). Mark the spots where the pits are to be excavated for the poles. Then erect the poles in a line (keeping them 104 *aṅgulas* apart) ". Kāt. Śl. 6.10.

Then with the measure of a *prakrama*, as per rule 9 above, construct a *vedi* whose width is 36 *prakramas*. Leave off a width of 12 *aṅgulas* from the East. Fix posts, whose diameter is 12 *aṅgulas*, at a distance of 104 *aṅgulas* from one another, in the North-South space in a line.

पार्श्वयोर्वाऽर्धमन्तर्वेदीति श्रुतेरर्धकानिति । का. शु. ६.११.

" One should erect, half the number of poles on each side half within the altar and half outside, so says the Śruti ". Kāt. Śl. 6.11.

The central pillar should be erected first. Then erect the five posts on the North of the central pillar and 5 posts on the South. Keep half of the width of 12 *aṅgulas* of each post inside and half outside, so that all the posts will be within the *vedi* and at the same time touching the *vedi* and just outside the *vedi*.

एके प्रथमोत्तमौ प्रकृतिवत् । का. शु. ६.१२.

" Some teachers prefer to erect only the first and the last one according to the instructions given for the construction of the basic *citi*. Kāt. Śl. 6.12.

Some *Ācāryas* prefer to erect the first and the last post inside the altar In the case of *stoma*, there is only one post, which is erected half inside and half outside. Similarly in the *vedi* having 11 posts the first and the last post must remain inside the *vedi* and the rest outside.

सैषा शिखण्डिनी वेदि: । का. शु. ६.१३.

" This *vedi* is called the *Śikhaṇḍinī vedi* ". Kāt. Śl. 6.13.

## *CHAPTER* VII—(Verses)

भवन्ति चात्र श्लोका:—

" There are a few verses, which explain the rules from Śulba ".

द्विहस्ते लक्षणं कुर्यात् त्रिहस्तो मध्यम: शिर: ॥ १ ॥
शिर:पश्चाद्वितस्ति: स्यात् पूर्वार्धे हस्त एव च ।
सार्धहस्ते च पाश: स्यात् वेदि: स्यात् पौर्णमासिकी ॥ २ ॥

" The cord, which has a mark at a distance of two hands or 48 *aṅgulas* from the end, its centre is at 72 *aṅgulas* or three hands from the end; there is a mark for the North-South side at a distance of 36 *aṅgulas* from this centre and the distance from this mark to the end of the cord including a sling is 36 *aṅgulas*, will be the measuring cord for the construction of an altar for the *Paurṇamāsa* sacrifice ". Kāt. Śl. 7.1-2.

There is another verse—which has a slightly different wording for the same altar, which is as under :—

पडरत्निर्द्विपाशा च मध्ये पञ्चसु चिह्निता ।
द्विहस्तेऽङ्गुलषट्के च त्रिष्वष्टादशकेषु च ।
सार्धहस्ते च पाश: स्याद्वेदि: स्यात् पौर्णमासिकी ॥ पृ. ४० हरिदास ग्रंथमाला

The measuring cord, which has slings at both ends should be 144 *aṅgulas* long and should have five marks at the following points. The first mark will be at 48 *aṅgulas* from one end for the *śroṇi* or for the North-South side. At 6 *aṅgulas* for the *nirañcana* and *śroṇi*, and 18 *aṅgulas* for the centre. The total length of the measuring cord from the end to the centre (which is 72), will be the length of the East-West line. On the other side at a distance of 18 *aṅgulas* from the centre, a mark is to be made for *aṁsa* and *nirañchana*. Again at 18 *aṅgulas* another mark for the North-South line on the East and at a distance of 36 *aṅgulas* will be the other end of the measuring cord including a sling.

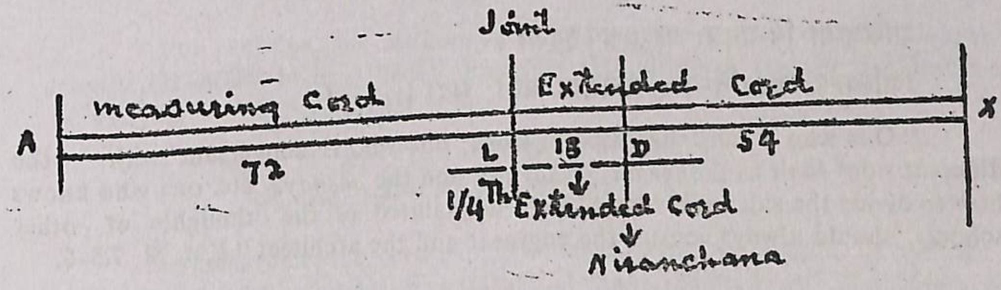

Fig 29

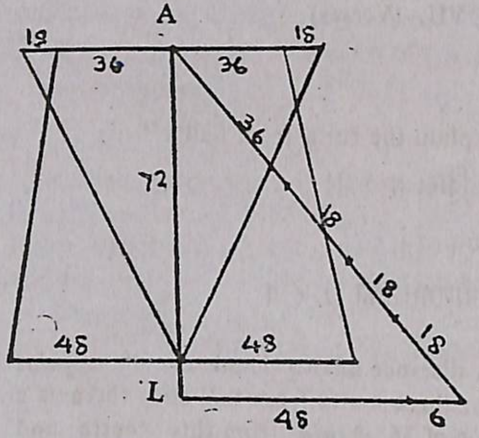

Fig 30

This *Vedi* has a shape of a trapezium,
AL = East-West line = 72.
Measuring cord + Extended Cord
= 144 *aṅgulas*.
*nirañchana* mark = $\frac{1}{4}$th of the extended cord = $\frac{72}{4}$ = 18 from the centre.

∴ The sides of the rt. ∠ed △ are 54, 72 and 90 i.e. the multiples of the rt. ∠ed △ of lengths of sides 3, 4 and 5.

संख्याज्ञः परिमाणज्ञः समसूत्रनिरञ्छकः ।
समभूमौ भवेद्विद्वाञ्छुल्बवित् परिपृच्छकः ॥ ३ ॥

"One who knows *Śulba*, must be well versed in numerals; a good surveyor even stretcher of the measuring cord; having knowledge of the plane ground and enquisitive". Kāt. Śl. 7.3.

न जलात् सममन्यत् स्यान्नान्यद्वातात् प्रमा भवेत् ।
नान्यद्दूरं भ्रमादूर्ध्वं नान्यत् सूत्रादृजुर्भवेत् ॥ ४ ॥

"There is nothing which remains in one plane like water; nothing is limited like wind; for a sane man nothing is far away; and nothing is straight like a measuring cord". Kāt. Śl. 7.4.

तिर्यङ्मान्याश्च सर्वार्थैः पार्श्वमान्याश्च योगवित् ।

"A *Śulba* scientist must have the full knowledge of both the sides : *tiryaṅmānī* and *pārśvamānī*" Kāt. Śl. 7.5.

करणीनां विभागज्ञो नित्योद्युक्तश्च कर्मसु ॥ ५ ॥
शास्त्रबुध्या विभागज्ञः परशास्त्रकुतूहलः ।
शिल्पिभ्यः स्थपतिभ्यश्च आददीत मतीः सदा ॥ ६ ॥

"One who is fully engrossed in work; one who is conversant with the different sides such as *dvikaraṇī*, *trikaraṇī*; and the *akṣṇayā* etc. one who knows how to divide the sides and who is well acquainted of the thoughts of other schools, should always consult the engineer and the architect". Kāt. Śl. 7.5-6.

Qualities of the wooden peg.

षडङ्गुलपरीणाहं द्वादशाङ्गुलमुच्छ्रितम् ।
जरठं चात्रणं चैव शङ्कुं कुर्याद्विशेषतः ॥ ७ ॥

द्विवितस्तिप्रमाणस्तु खादिरो मुद्गरस्तथा ।
शङ्कुस्तेन निखातव्यस्तस्मात्तस्य परिग्रहः ॥ ८ ॥
एकतस्तु ऋजुस्तीक्ष्णः खादिरः सममायतः ।

"An intelligent person, should prepare a peg from a log of wood, which is old and free from knots. The peg should be 12 *aṅgulas* in length and 6 *aṅgulas* in circumference. A mallet, of the length of two *vitastis* or 24 *aṅgulas* prepared out of strong wood (*khadira*) is always necessary as the same is used for driving a peg. One having a thorough knowledge of Śulba, should prepare a peg out of strong wood (*Khadira*), one end of which is plain and the other straight, long and tapered". Kāt. Śl. 7.7-9.

शङ्कुः कार्यस्तु शुल्बज्ञैस्तस्यार्धं गमयेन्महीम् ॥ ९ ॥
प्रादेशमात्रो हविर्यज्ञे पूर्वलक्षणलक्षितः ।
शङ्कुरामशिराः कार्यस्तस्याप्यर्धं निखापयेत् ॥ १० ॥
चतुरस्रमुद्गरं स्यात् षोडशाङ्गुलमायतम् ।
अविद्धं रमणीयं च दारुमध्याच्च निर्मितम् ॥ ११ ॥

"In the case of sacrifices, a peg, having the above qualities, should be prepared from a piece of green wood. The length of the peg should be 9 *aṅgulas*. Half of this length should be buried inside the ground".

"A mallet, which is in one piece and square in shape and which would drive the peg nicely, should have a length of 16 *aṅgulas*. This should be prepared from a piece of wood obtained from the centre of a log". Kāt. Śl. 7.9-11.

The qualities of the Cord.

अजीर्णाऽग्रन्थिनी सूक्ष्मा समा श्लक्ष्णा त्वरोमशा ।
रज्जुर्मानाधिका कार्या अध्वरे योगमिच्छता ॥ १२ ॥

"In a sacrifice, the *Adhvaryu* should prepare a cord which is longer in length. It should be new, free from knots, thin, uniform, smooth and should not be hairy". Kāt. Sl. 7-12.

शाणी वा बाल्वजी चैव वैणवी वा विधीयते ।
रज्जुस्तूभयतःपाशा त्रिवृता यज्ञकर्मणि ॥ १३ ॥

"In a sacrifice, the cord should be three stranded, should have slings at both ends, should be longer than the actual length required and should be prepared from hemp, grass and bamboo". Kāt. Śl. 7.13.

रज्जुर्मुञ्जमयी कार्या शणैस्तु परिमिश्रिता ।
कात्यायनो वदत्येवमखण्डा कुशबल्बजैः ॥ १४ ॥

" *Ācārya Kātyāyana* says that the measuring cord should always be in one piece and should be prepared from *muñja* grass mixed with hemp, *darbha* and *balbaja* grass ". Kāt. Śl. 7.14.

नवके लक्षणं कुर्यात त्रीणि कुर्यात् त्रिषु त्रिषु ।
उत्तमो नवकः पाशः सदसो मानमुच्यते ॥ १५ ॥

" Make a mark on the nine and every third part. The ends should be provided with slings. These are the measurements for the *sadas* ". Kāt. Śl. 7.15

This Hall named *sadas* is erected facing North.[1]

*Prāgvaṁśa* is a square.[2]

*Śālā* is rectangular : Length 20 *aratnis* and width 10 *aratnis*.[3]

*Vimita* is a hall constructed at the *Soma* sacrifice. It is square in shape, its side is 10 *aratnis*.[4]

A clear and simple description of the above is given as follows :—

पञ्चदशमथैकविंशतिकमपरं परस्त्रिकं च ।
द्वादशसु पाश उत्तम इति सोमे रज्जुमानमेतत् ॥ १६ ॥

" In the case of *Soma* sacrifice, a measuring cord should have marks at 15; then at 21; then at 3; again at 3 and at the end at 12 with a sling ". Kāt. Śl. 7.16.

The marks should be at 15, for the *śroṇī nirañchana* from there at 21 for the centre, then at 3 for the *aṁsa nirañchana* again at 3 for *aṁsa* and then at 12 *prakramas* including, the sling. (as the East side is 24 *prakramas*).

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(1) सोमयागे सदोऽभिधानो मण्डपोऽष्टादशारत्निदीर्घो नवारत्निविस्तृतो दीर्घचतुरस्र उदग्वंशो विहितः । का. श्रौ. सू. ९.६.३.

तत्रापि शालावत् उदीची प्राचीत्वेन व्यवहर्तव्या । सदोमण्डपस्य दैर्घ्यें विस्तारे चान्येऽपि पक्षाः सन्ति ते तत एवावगन्तव्याः । का. श्रौ. सू. ९..६.६.

सदोमण्डपः दैर्घ्ये अष्टादशारत्निः विस्तारे नवारत्निः तथा च मुख्यमण्डपस्याभ्यन्तर एव पूर्वभागे भवति ।

(2) समचतुरस्रं प्राग्वंशं भवति । का. शु. महीधर भाष्य पृ. ७.

(3) विंशत्यरत्निः शाला स्यात्तदर्धेन तु विस्तृतः । का. शु. पृ. ७.

(4) विमितं चतुरस्रं स्याद्दशारत्निप्रमाणतः । का. शु. पृ. ७.
विमितम् इति मण्डपविशेषः सोमयागे प्रसिद्धः । का. शु. पृ. ७.

पदस्याक्ष्णया तिरश्ची तयोरक्ष्णया भवेत् ।
सौत्रामण्यां विमातव्या वेदिः स्यात् सोमवत्तया ॥ १७ ॥

" Take the hypotenuse of a rt. ∠ed △ whose sides are a *pada* each, a one side of another rt. ∠ed △ and the other side as one *pada*, then the hypotenuse of this rt. ∠ed △ will be the measure of a *prakrama* for the *Sautrāmaṇī vedi* ". Kāt. Śl. 7.17.

नीहारेण घनैर्वापि ज्योतिषामभ्रदर्शने ।
अप्सु दीपं प्रगृह्णीयाद्यावत्तमसि दर्शने ॥ १८ ॥

" In case, if the stars are not visible due to mist or clouds one should hold a lamp in water', as long as the darkness exists ". Kāt. Śl. 7.18.

प्रमाणं च प्रमेयं च यच्चान्यद्वस्तुसंज्ञकम् ।
सर्वं तच्छास्त्रतो ज्ञात्वा यज्ञे सिध्यन्ति याज्ञिकाः ॥ १९ ॥

" The sacrificers should conduct their sacrifices, with a thorough knowledge of the rules described in the sacred books regarding that what is to be measured and its scale etc. ". Kāt. Śl. 7.19.

यथा न क्षीयते मानं यथा च न विवर्धते ।
यथा च रमते दृष्टिस्तथा योगं समाचरेत् ॥ २० ॥

" Measurements should be carried out in such a way that the actual measure will neither be less nor more and with reasonable means as gaze will settle on it ". Kāt. Śl. 7.20.

अरत्निश्चतुरस्रस्तु पूर्वस्याग्नेः खरो भवेत् ।
रथचक्राकृतिः पश्चाच्चन्द्रार्ध इव दक्षिणः ॥ २१ ॥

" A square of 24 *aṅgulas*, on the East will form the altar of the *Āhavanīya* fire. The *Dakṣiṇāgni* is semi-circular. The *Gārhapatya* fire on the West, will be circular ". Kāt. Śl. 7.21.

अग्नीनां तु खरः कार्यो मेखलात्रयसंयुतः ।
द्वादशाङ्गुल उच्छ्राये विस्तारे चतुरङ्गुलः ॥ २२ ॥

" For receiving the fire, the mound should be of 3 pedestals, 4 *aṅgulas* in breadth and 12 *aṅgulas* in height ". Kāt. Śl. 7. 22.

तन्तुः पुष्करनालस्य षड्गुणः परिवेष्टितः ।
वत्सतर्यास्त्रिहायण्या बालेन सदृशो भवेत ॥ २३ ॥

" The thickness of the hair of a three year old calf is equal to six times the fibre of a stem of a lotus ". Kāt. Śl. 7.23.

त्रयस्त्रिहायणीबालाः सर्षपार्धं प्रचक्षते ।
द्विगुणं सर्षपं विद्याद्यवः पञ्च तु सर्षपाः ॥ २४ ॥

"The thickness of the three hair of a three year old calf are equal to the thickness of half a mustard seed. That is a mustard seed is twice that of the thickness of three hair of a three year old calf. Five mustard seeds equal to one *yava*". Kāt. Śl. 7.24.

अङ्गुलस्य प्रमाणं तु षड् यवाः पार्श्वसंस्थिताः ।
चतुर्विंशाङ्गुलोऽरत्निर्वितस्तिर्द्वादशाङ्गुला ॥ २५ ॥

"Six *yavas*, when placed side by side with their widths touching one another are equal to one *aṅgula*. 24 *aṅgulas* make one *aratni* and 12 *aṅgulas* are equal to one *vitasti*". Kāt. Śl. 7.25.

व्यामस्यात्र प्रमाणं तु चतुर्न्यूनं शतं भवेत् ।
पुरुषस्य प्रमाणं वै विंशतिस्तु शताधिका ॥ २६ ॥

"The measure of a *vyāma* is 4 *aṅgulas* less than 100, that is equal to 96 *aṅgulas* and the measure of a *puruṣa* is 120 *aṅgulas*". Kāt. Śl. 7.26.

हिरण्यशकलार्थे तु हिरण्यं यस्य नोच्यते ।
कृष्णलेनैव तद्व्याख्या यज्ञे सिध्यति याज्ञिकी ॥ २७ ॥

"Red peas are offered at the time of sacrificial work. These though not of gold serve its purpose". Kāt. Śl. 7.27.

कृष्णलं त्रियवं मानं ताम्रायसमतः परम् ।
सुवर्णादर्धं च माषाणां सुवर्णाश्च त्रिसप्ततिः ॥ २८ ॥
त्रीणि चैव सहस्राणि दद्याद्बहुसुवर्णके

"One red pea is equal to 3 *yavas* (in weight). The next weight is of copper. In the case of *Bahusuvarṇaka* distribute 3073 gold pieces and half the ldgo in māṣas". Kāt. Śl. 7.28–29.

भूयः स्यपतितो ज्ञात्वा संज्ञास्वन्यासु मानवित् ।
स्वर्णकारो यथाऽभ्यासात्तथा भूयो विवर्धते ॥ २९ ॥

"One anxious to know about bigger measurements than these should approach the goldsmith or an engineer". Kāt. Śl. 7.28–29.

ह्रसते शोषपाकाभ्यां द्वात्रिंशद्भागमिष्टका ।
तस्मादार्द्रप्रमाणं तु कुर्यान्मानाधिकं बुधः ॥३०॥

"Bricks decrease (in length, breadth and thickness) by $\frac{1}{32}$ while drying and burning. Therefore, a wise man should always keep these measurements slightly in excess". Kāt. Śl. 7.30.

अज्ञात्वा शुल्बसद्भावं यज्ञे सौत्रामणीसुते ।
वेदिं ये कर्तुमिच्छन्ति गिरिं भिन्दन्ति ते नखैः ॥ ३१ ॥

"In the case of sautrāmaṇī sacrifice, if one constructs the fire-altar, without the proper knowledge of the rules of *Sulba*, he will be doing the same as if digging a mountain with nails". Kāt. Śl. 7.31.

दण्डरज्ज्वर्धमभ्यस्य षष्ठे त्वर्धस्य लक्षणम् ।
तथैव चेतरत्रापि तिर्यङ्मानं यदृच्छया ॥ ३२ ॥

"Having increased the measuring cord by half its length, make a mark of *nirañchana* on the sixth part of the increased cord. Mark similarly the sign of *nirañchana* on the other side also. The length of the North-South line should always be kept longer or smaller as one desires". Kāt. Śl. 7.32.

यावत्प्रमाणा रज्जुः स्यात्तावानेवागमो भवेत् ।
आगमार्धे भवेच्छङ्कुस्तदर्धे च निरञ्छनम् ॥ ३३ ॥

"Having increased the measuring cord by its own length, make a mark on half the length of the increased cord for *śroṇī* and *aṁsa* and fix the pegs accordingly. Similarly make a mark on half of the halved part of the cord for *nirañchana*" Kāt. Śl. 7.33.

आधाने पदिकं कुर्यात् द्विपदः सौमिको भवेत् ।
अग्नौ च त्रिपदं कुर्यात् प्रक्रमं याज्ञिको बुधः ॥ ३४ ॥

"A *prakrama* should be of the length of a *pada*, when the ceremony for establishing the perpetual sacred fire is being performed. The measure of a *prakrama* will be of two *padas*, when an altar is to be constructed :for the *Soma* sacrifice; and the measure of a *prakrama* will be of three *padas*, when the altar of the *Mahāvedi* is to be constructed". Kāt. Śl 7 34.

कृत्तिका श्रवणः पुष्यश्चित्रास्वात्योर्यदन्तरम् ।
एतत्प्राच्या दिशो रूपं युगमात्रोदिते पुरः ॥ ३५ ॥

"*Krittikā*, *Śravaṇa*, and *Puṣya* all these stars rise on the East. The Sun always rises between the distance of *citrā* and *svātī*, while these stars are coming up the horizon by about 86 *aṅgulas* the East-West line should always be demarcated". Kāt. Śl. 7.35.

पञ्चाशच्छर्कराः पश्चात् पूर्वे देयास्त्रिसप्ततिः ।
दक्षिणे तु प्रदातव्या दश पञ्च च सप्त च ॥ ३६ ॥

"(According to the *Kāṭhaka cayana*), 50 sand pebbles on the West, 73 on the East and 10, 5 and 7 are to be placed on the South". Kāṭ. Śl. 7.36.

*Kāṭhaka Cayana* is explained in the *Taittirīya Brāhmaṇa* (3.10–12) and in the *Taittirīya Āraṇyaka* (1 and 3). These *Cayanas* are called *Sāvitra*, *Nāciketa Cāturhotra*, *Vaiśva-sṛja* and *Āruṇa-ketuka*, The procedure to be followed in each one of them differs. These are performed along with the *Soma* sacrifices. In the above *Cayanas* small pebbles are used instead of bricks. Small pieces of gold of the size of the toe of one's finger can be used in the place of the sand pebbles. The shape of all these *Cayanas* is circular.

In the *Sāvitra Cayana*, a circle of the size of the wheel of a chariot is first drawn and inside this circle, nine other circles not overlapping but of different radia are drawn and on these circles small sand pebbles are placed.

The procedure to be followed in the case of *Āruṇaketu* is similar to that of the *Cayana* of Sāvitra, but water is used in place of bricks. These water-bricks are made of river water and other sacred waters, which are either hot or cold.

शंस्यश्चतुर्विंशतिपार्श्वभागश्चतुर्दशभिः परिलेख्यरतु नर्यम् ।
तथैव चाष्टद्विगुणैरथर्य्यास्त्रिंशद्भिरायम्य हरेत्तृतीयम् ।। ३७ ।।

" *Āhavanīya* is a square of 24 parts situated on the East, *Gārhapatya* is a circle of 14 parts on the West and the *Dakṣiṇāgni* on the South of 16 parts". Kāt. Śl. 7.37.

अग्नेरुदक्सार्धनवाङ्गुले मध्यं ततो लिखेत् ।
वृत्तमेकोनविंशत्या प्राचीज्या मध्यगा भवेत् ।

"From the place of *Dakṣiṇāgni*, mark the centre of a circle $9\frac{1}{2}$ *aṅgulas* towards North. Draw a circle with this as centre and radius of $9\frac{1}{2}$ *aṅgulas*. The East-West line will be the diameter, then cut off the northern part and make a semi-circular mound on the south of one *aratni* ". Kāt. Śl 7.38.

उदगर्धं विहायार्त्नीक् खराग्नेर्दक्षिणस्य तु ।। ३८ ।।
सूत्रदोषदरिद्रस्य गूढमन्त्रस्य धीमतः ।
समाप्तेयं क्रिया शौल्बी कात्यायनमहात्मनः ।। ३९ ।।

" Ācārya *Kātyāyana* the talented has finished this Śulba, the rules therein are faultless and their meaning in concealed ". Kāt. Śl. 7.38–39.

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# SECTION II

## *ARTICLES*

*related to*

## KĀTYĀYANA ŚULBA SŪTRA

# CHAPTER I

## The Steps leading to the growth and development of the Theorem of the hypotenuse (now commonly known as the Theorem of Pythagoras in India)

In this article, an attempt has been made to trace the various steps, which led the *Sūtrakāras* to the discovery of the Theorem of the hypotenuse.

This development took place in three different ways :

(1) Fixation of the East-West line, the points of the *śroṇīs* and the *aṁsas.*

(2) The knowledge of the two sets of the wellknown rt. ∠ed △s having sides of lengths of 3, 4 and 5 units and having sides of lengths of 5, 12 and 13 units or their multiples having sides of lengths of 15, 39, and 39 units.

This was further followed with the additions of a few more sets of rt ∠ed △s.

(3) But when confronted, with the construction of new altars, where the above two methods were found to be inadequate, there was no other way left out than to provide a general statement of the above theorem.

Now let us deal with the above developments one by one :

I. This Method is described in detail by *Kātyāyana* and the same will be reproduced here in his own words.

प्राच्यन्तयोः शङ्कू निहन्ति । का. शु. १.६.

" One should fix a peg at each end of the East-West line. " (This distance between the two pegs must be equal to the measuring cord). Kāt. Śl. 1.6.

श्रोण्योरंसयोश्च । का. शु. १.७.

" One should fix pegs at the points of both *śroṇīs* and *aṁsas.* " Kāt. Śl. 1.7.

*Śroṇī* = North-West and South-West corners. *Aṁsa* = North-East and South East corners.

शङ्कोः पाशौ प्रतिमुच्य निरञ्छनेन गृहीत्वा दक्षिणपूर्वां दिशं हरन्ति । का.शु. १.८.

Having fixed slings, on both the pegs of the East-West line, one shoud stretch the cord (which is already doubled) by the *nirañchana* mark to the South-East and fix a peg (at a point where the *nirañchana* mark meets the ground) ". Kāt. Śl. 1.8.

एवमुत्तरतः । का. शु. १.९.

" The same procedure should be followed towards the North ".

Kāt. Śl. 1.9.

*Nirañchana* is a name given to a mark made on the measuring cord, which enables one to lay out a rt. ∠ed △ on the ground. Kāt. Śl. 1.9.

विपर्यस्येतरतः । का. शु. १.१०.

" Having exchanged the slings one should reverse the process on the other side ". (i.e. on the West). Kāt. Śl. 1.10.

*Ācārya Kātyāyana* asks, as per above rules, to mark the East-West line first. And then to drive pegs on this line at a measured distance apart which is equal to the length of the measuring cord and then fix the pegs at the points of *śroṇīs* and *aṁsas*, by stretching lines at right angles to the East–West line, from the pegs at the extreme ends. Once this is completed, he should adopt the following procedure :

प्रमाणार्धे समचतुरस्रस्य शङ्कुः । का. शु. १.१६.

" At the time of fixing the corners of the square, one should drive a peg at half the length of the measuring cord ". Kāt. Śl. 1.16.

शास्त्रवदर्धे दीर्घचतुरस्रस्य । का.शु. १. १७.

" In the case of a rectangle, one should drive a peg at half the measure of the North-South line (*tiryaṅmāṇī*), as prescribed in the holy text ". Kāt. Śl. 1.17.

शकटमुखस्य चैवम् । का. शु. १.१८.

" One should follow the same procedure (as described above) in the case of an altar, the shape of which is just like the fore part of a chariot ". Kāt. Śl. 1.18.

*Prauga* = An isosceles triangle.

एतेन प्राग्वंशवेदिमानानि व्याख्यातानि । का. शु. १.१९.

" Here the measurements (constructions) for the *Prāgvaṁśa, Havirdhāna* and such other pandals along with different shapes of altars should be considered as explained ". Kāt. Śl. 1.19.

From the above four *sūtras*, it will be clear that marks for *śroṇīs* and *aṁsas* should be provided as prescribed, thus in the case of a square, it will be half the measuring cord; in a rectangle, it will be half of the North South line; in the case of a triangle. there will be no mark for the *aṁsa* but for fixing the *śroṇīs* it will be half of the North-South line. Thus the East–West line forms the main central axis and by fixing the points of *śroṇis* and *aṁsas*, (which are equal to half the lengths of the other two sides) and altar having rectilinear figure can be easily constructed. Take the case of *Mahāvedi*. It has a shape of a trapezium. The two parallel sides are of 24 and 30 units and the distance between them is of 36 units in length.

First mark the East West-line of 36 units (AB).

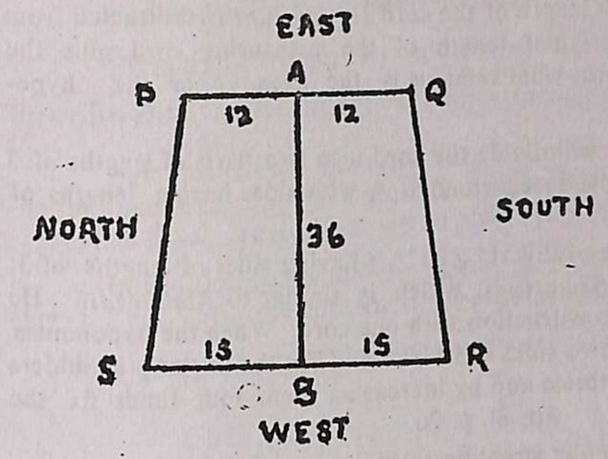

Fig 31

With half the length of 24 i.e. $\frac{24}{2} = 12$, mark points P and Q at right angles to AB at A. These points denote the *aṁsas*. Similarly mark points R and S or *śroṇīs* at a distance of 15 units $\left[\frac{30}{2} = 15\right]$ at right angles to AB at B. Thus PQRS will form the outer boundary of the *Mahāvedi* required. All the points P, Q, R and S are at right angles to AB and are fixed with the help of right angles led out with the half of the *nirañchana* mark. Thus it will be quite clear that a *vedi* with a rectilinear figure can be constructed when—

(1) the East–West line is fixed, and

(2) the points *śroṇīs* and *aṁsas* at right angles to the East-West line are fixed.

To construct a rt. angle with the help of *nirañchana* mark.

II. (A) The first chapter of the *Kātyāyana Śulba Sūtra* is *Paribhāṣā-Prakaraṇa*. *Paribhāṣā* means definitions of technical terms, which are applicable throughout the text. Here we are mainly concerned with the definitions of the words *nirañchana* and *akṣṇayā*.

It should be noted that for the word *nirañchana*, there are two separate definitions and have two separate conclusions. These definitions are quoted by all the Śulbakāras. Similarly the word *akṣṇayā* (hypotenuse) which is the result based on the above definitions is the same and is repeated twice. Let us see what these definitions are and what they actually mean.

The first definition is as under :

प्रमाणमभ्यस्याभ्यामचतुर्थे लक्षणं करोति तन्निरञ्छनम् । का. शु. १.१२.

(अ) यावत्प्रमाणा रज्जुः स्यात्तावानेवागमो भवेत् ।
आगमार्धे भवेच्छङ्कुः तदर्धे च निरञ्छनम् ।। का. शु. ७.३३.

" Having increased the length of the measuring cord by its own length, one should make a mark on the fourth part of the increased cord (which is nearer to the junction of the original measuring cord and the increased cord). This mark is termed as the *nirañchana*. " Kāt. Śl. 1.12.

अक्ष्णया तिर्यङ्मानीशेषः । का. शु. १. १३.

" When the North-South length of the cord *tiryanmānī* is subtracted from he total length of the cord, (original length of the measuring cord plus the extended length of the same cord) what remains is the *akṣṇayā* or the hypotenuse ". Kāt. Śl. 1.13.

Thus the *nirañchana* mark will divide the cord into two parts of lengths of 3 and 5 units and will be able to produce a rt. ∠ed △ with sides having lengths of 3, 4 and 5 units.

Āpastamba has made use of this rt. ∠ed △ ( having sides of lengths of 3; 4 and 5 units in constructing a *Soma-vedi*, which is similar to *Mahāvedi*. He says :—"This is the method of construction with one cord. When the hypotenuse of a rt. ∠ed △ is 5, the other two sides are 3 and 4. Fix the eastern shoulders by increasing the original sides thrice and by increasing them four times fix the western shoulders.[1] (*śroṇi*) ". Āp. Śl. p. 76.

II. (B) Now let us turn our attention to the second definition of *nirañchana*. It is as follows :

प्रमाणार्धं वाऽभ्यस्याभ्यासषष्ठे लक्षणं करोति तन्निरञ्छनम् । का. शु. १. १४

(अ) अक्ष्णया तिर्यङ्मानीशेषः । का. शु. १.१५.

(ब) दण्डरज्ज्वर्धमभ्यस्य षष्ठे त्वर्धस्य लक्षणम् ।
तथैव चेतरत्रापि तिर्यङ्मानं यदृच्छया ।। का. शु. ७.३२.

" Having extended the measuring cord, by its half, one should make a mark, on the sixth part of the extended (half) cord. (This mark should be nearer to the junction of the original and the extended cord.) This mark is termed as *nirañchana*. " Kāt. Śl. 1.14.

*Nirañchana* is nothing but a mark or a sign made at the junction of the shorter side and the hypotenuse.

Thus this cord will be suitable for a rt. ∠ed △ whose sides are 5 and 12 and the hypotenuse of 13 units.

This rt. ∠ed △ was made use of in the construction of the *Mahāvedi*. The measurements of the *Mahāvedi* are :

स वेद्यन्तात् षट्त्रिंशत्प्रक्रमान् प्राचीं वेदिं विमिमीते त्रिंशत् पश्चात्तिरश्चीं चतुर्विंशतिं पुरस्तात्तन्नवतिः । सैषा नवतिप्रक्रमा वेदिस्तस्यां सप्तविधमग्निं विदधाति । शत.ब्रा.१०.२.३.४

" From the western end of the *vedi*, he measures off the *vedi*, 36 steps long eastward, 30 steps broad behind and 24 steps broad in front. The total of these three measures makes 90. This then is the *vedi* measuring 90 steps. Thereon he lays out the seventh fold fire-altar. " Śata. Brāh. 10.2.3.4.

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[1] तदेकरज्ज्वा विहरणम् । त्रिकचतुष्कयोः पञ्चिकाऽक्ष्णया रज्जुः । ताभिस्त्रिरभ्यस्ताभिरंसौ । चतुरभ्यस्ताभिः श्रोणी । आप. शु. पृ. ७६.

Similar description is given in the *Taittirīya Saṁhitā*.

त्रिंश्शत्पदानि प्रक्रमा वा पश्चात्तिरश्ची भवति, षट्त्रिंशत् प्राची चतुर्विंश्शतिः पुरस्तात्तिरश्चीति महावेदेर्विज्ञायते, मानयोगस्तस्या व्याख्यातः । बौ. शु. १.९०.

" As this earth is the *vedi*, they can measure off and sacrifice on so much as they can use. The back cross-line is 30, the eastern line is 36 and the front cross line is 24. These make up 90. " Tait. Saṁ. 6.2.4.5; Baudhā. Śl. 1.90.

In addition to the above rt. ∠ed △s. Āpastamba makes use of the following rt. ∠ed △s for the construction of the *Mahāvedi*.

द्वादशिकापञ्चिकयोस्त्रयोदशिकाऽक्ष्णयारज्जुः ताभिरंसौ द्विरभ्यस्ताभिः श्रोणी ।
आप.शु.पृ. ८१.

" The hypotenuse is 13 (of a rt. ∠ed △, the lengths of whose sides are of 5 and 12 units). The eastern corners are decided by increasing the original lengths of the sides twice (i.e. making the sides three times the original length ) " Āpa. Śl p. 81.

पञ्चदशिकाऽष्टिकयोः सप्तदशिकाऽक्ष्णयारज्जुः । ताभिः श्रोणी । आ.शु.पृ. ८१.

" The hypotenuse is 17 (of a rt. ∠ed △) the lengths of whose sides are of 8 and 15 units. This is useful for fixing the western corners. " Apa. Śl. p.81.

द्वादशिकापञ्चत्रिंशिकयोस्सप्तत्रिंशिकाऽक्ष्णया रज्जुः ; ताभिरंसौ । आप.शु. पृ.८२.

" The length of the hypotenuse is 37, when the lengths of the sides of the rt. ∠ed △ are 12 and 35 units. This is useful for fixing the eastern corners." Āpa. Śl. p. 82.

Similarly, Baudhāyana gives a list of sides with whose help rt. ∠ed △s can be prepared; but these are stated without stating the lengths of the hypotenuses. These are mentioned as illustrative examples or as a corolary after stating the property of the main theorem which is :

दीर्घचतुरस्रस्याक्ष्णया रज्जुस्तिर्यङ्मानी पार्श्वमानी च यत्पृथग्भूते कुरुतस्तदुभयं करोतीति
क्षेत्रज्ञानम् । का.शु. २.११.

(अ) दीर्घचतुरस्रस्याक्ष्णया रज्जुः पार्श्वमानी तिर्यङ्मानी च यत्पृथग्भूते कुरुतस्तदुभयं
करोति । बौ. शु. १.४८.

टीका :—दीर्घचतुरस्रपार्श्वमान्या समचतुरस्रे कृते यत् क्षेत्रं संपद्यते यच्च तिर्यङ्मान्या तदुभयमक्ष्णयारज्ज्वा समचतुरस्रे कृते संपद्यते । त्रिकचतुष्कयोः पञ्चिकाक्ष्णयारज्जुरित्याद्युदाहरणम् । तासां त्रिकरणेऽप्ययमेव प्रकारः ।

The square on the hypotenuse is equal to the sum of the squares on the other two sides. Kāt. Śl. 2.11, Baudhā. Śl. 1.48.

The list of rt. ∠ed △s as quoted by *Baudhāyana* is as follows :

त्रिकचतुष्कयोर्द्वादशिकपञ्चिकयोः पञ्चदशिकाष्टिकयोः सप्तिकचतुर्विंशिकयोर्द्वादशिक-पञ्चत्रिंशिकयोः पञ्चदशिकषट्त्रिंशिकयोरित्येतासूपलब्धिः । बौ. शु. १.४९.

"This is seen in those rt. ∠ed △s the sides of which are ; (a) 3 and 4 (b) 12 and 5 (c) 15 and 8, (d) 7 and 24, (e) 12 and 35 and (f) 15 and 36 ".

*Baudhāyana* quotes 7 sets of rt. ∠ed △s, whilst *Āpastamba* quotes only six. The important point to be noted is that all these sets can be used for the construction of the *Saumiki vedi* or *Mahāvedi*.

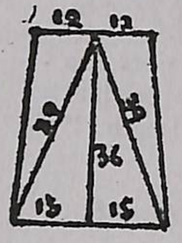

Fig 32

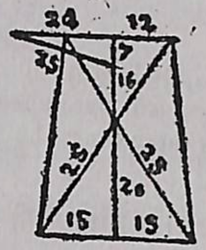

Fig 33

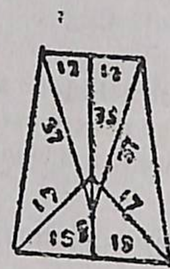

Fig 34

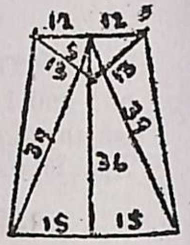

Fig 35

In addititon to the above there are two more sets of rt. ∠ed △s, the lengths of whose hypotenuses are quoted in *sūtras* These are :

पदं तिर्यङ्मानी त्रिपदा पार्श्वमानी तस्याक्ष्णयारज्जुर्दशकरणी । का. शु. २.८.

" In a rt. ∠ed △ whose side is one *pada* and the other side is three *padas* its hypotenuse will produce an area of ten sq. *padas* ". Kāt. Śl. 2.8.

एवं द्विपदा तिर्यङ्मानी षट्पदा पार्श्वमानी तस्याक्ष्णया रज्जुश्चत्वारिंशत्करणी ।

का. शु. २.९.

" In a rt. △ed △ whose one side is two *padas*, and the other side six *padas*, the hypotenuse will (be a *tat-karaṇī* of a square) produce an area of 40 sq. *padas* ". Kāt. Śl. 2.9.

Conclusions : (1) **Out of all these rt. ∠ed △s quoted above the definitions for the mark of the nirañchana are available for the construction of two rt. ∠ed △s only,** as these rt. ∠ed △s were known to the sacrificers for a very long time. These rt. ∠ed △s are 3, 4, 5 and 5 12 and 13.

(2) It was not a very difficult task for the *Sūtrakāras* to go in for a definition of the *nirañchana* mark, for every new set of a rt. ∠ed △. But why this method was not followed.

(3) Because this method was found to be most unwieldy.

(4) It would have been very difficult for the sacrificer to remember each case separately and then make use of the *nirañchana* mark. As with the knowledge of every new rt. ∠ed △, a separate and a new definition for the mark of *nirañchana* would have been necessary.

(5) As long as the lengths of the sides were in rational numbers, the definition for the *akṣṇayā* or the hypotenuse was a check, but when irrationals for the values of the *akṣṇayā* or the hypotenuse came in, the problem became more difficult.

(6) The general enunciation based on mathematical truths must have been arrived at and hence there was no necessity of defining both the *nirañchana* and the *akṣṇayā* or the hypotenuse.

(7) Thus the knowledge of the rt. ∠ed △s, 3, 4 and 5 and 5, 12 and 13 along with a few others was made use of in the construction of altars at this stage. (See sketches of *Mahāvedi* shown above).

From the above illustrations it will be clear that the ancient Indians had clearly acquired the knowledge of the following :

(1) They knew how to construct a rt. ∠ed △ when the sides were known.

(2) In case of a rt. ∠ed △ the property of the hypotenuse and the other two sides was properly understood.

III. Let us now find out—as to what led these people to go in for the general definition and how far they succeeded in this attempt.

(A) The first sacrificial altar to be constructed was a sq, of 7½ sq. *puruṣas.*

उत्तरेषु पुरुषोच्चयेनैकशतविधादित्येतद्वक्ष्यामः ।

का.शु. ५.१. का. श्रौ. सू. १६.८.२७.

(अ) तदुह वै सप्तविधमेव चिन्वीत । सप्तविधो वाव प्राकृतोऽग्निः ।
तत ऊर्ध्वमेकोत्तरानिति विज्ञायते । आप. शु. पृ. १३७.

(ब) अर्धाष्टमपुरुषाः प्रथमोऽग्निः । अर्धनवमा द्वितीयः । अर्धदशमास्तृतीयः ।
एवमुत्तरोत्तरो विधाभ्यास आ एकशतविधात् । बौ. शु. २.(१-४).

" The increase of a square *puruṣa*, in every successive fire-altar starting from the sevenfold altar to the hundred and one fold fire-altar. " Kāt. Śl. 5.1. Kāt. Śr. Sū. 16.8.27. Āpa. Śl. p. 137, Baudhā. Śl. 2.(1–4.)

The above statement contained in the Kāt. Śr. Sū. 16.8.27 shall be explained now.

Thus it will be quite clear from the statement quoted above (similar statements are made by *Baudhāyana* and *Āpastamba* which are already quoted) that at the time of constructing a new fire-altar, if the first sacrifice (*saptavidha* = sevenfold fire-altar) is already performed, area of one square *puruṣa* was to be added to the previous altar in the form of a square.

**Thus the addition of two squares having unequal areas was a necessity**

**(B) Similarly-when the shape of the altar was divided into different squares such as : *Ātman*; Wings and tail, the difference or subtraction of two squares was found necessary.**

This addition or subtraction which is found out by all the *Sūtrakāras* in exactly the same way is based on the theorem of the Hypotenuse.

**(C) All the *Sūtrakāras* agree that the altar for the *Aśvamedha* sacrifice must be twice or three times the area of the basic fire-altar : (sum of two or three equal squares).**

(D) Sum of two equal circles required in constructing *Dakṣiṇāgni*.

आद्योऽग्निर्द्विगुणस्त्रिगुणो भवतीति सर्वसमासः । का. शु. ५.२.

(अ) द्विस्तावा वेदिर्भवतीत्यश्वमेधे विज्ञायते । आप. शु. पृ. ८६.

(ब) त्रिस्तावोऽग्निर्भवतीत्यश्वमेधे विज्ञायते । आ. शु. पृ. ८६.

(क) वेदिकाले द्विस्तावा वेदिः । त्रिस्तावोऽग्निरेकविंशो वा । आप.श्रौ.सू. नवमी कण्डिका.

(ड) तावत्परेण मध्यतो देवयजनं जोषयते द्विस्तावद्यथाग्नेर्विधायाम् । बौ.श्रौ.सू. १५.१

"The basic *citi* should be doubled or tripled (in area)". Kāt. Śl. 5-2.

This is an example of the addition of two or three equal squares :

If the side of a square or the dia of the circle which is likely to produce an area which is twice or three times the area of the original square or the circle, then its side is supposed to have some relation with the side of a square of the original basic *citi* or the dia of the circle. Thus :

(1) The side of the new square or the dia of the circle will not be exactly double or thrice the length of the side of the square of the basic *citi* or the dia of the original circle.

(2) The solution of this problem is not possible with the help of the rt. ∠ed Δs that were known,

In order to arrive at the solution, therefore, one must be thoroughly conversant with the main principles of the theorem of the hypotenuse. Without its application this problem cannot be solved.

(D) (1) The area of the *Sautrāmaṇī vedi* is one third of the area of the *Mahāvedi*.

वितृतीयं वै सौत्रामणी, वितृतीये यजेत , प्रक्रमतृतीयेनावृत्तेन इति सौत्रामण्यामुक्तः । का.श्रौ. सू. १९.२.२.

(अ) सौमिक्या वेदेर्वितृतीयदेशे यजेतेति सौत्रामण्या वेदेर्विज्ञायते । आप.शु.पृ.८५.

(ब) वेदितृतीये यजेतेति सौत्रामणिकीं वेदिमभ्युपदिशन्ति । बौ. शु. १.८१.

" He is to sacrifice on the third part of the *vedi*, thus they teach concerning the *vedi* of the *Sautrāmaṇī* sacrifice. " Kāt. Śr. Sū. 19.2.2.

(2) Similarly the *vedi* of the *Pitṛyajña* will have an area equal to the ninth part of the area of the *Mahāvedi*.

त्रितृतीया वेदिर्भवतीति पैतृक्या वेदेर्विज्ञायते। बौ. शु. १.८१.

" The *vedi* is to be formed with the third part, that is the tradition for the *vedi* of the *Pitṛyajña*". Baudh. Śl. 1.81.

महावेदेस्तृतीयेन समचतुरस्रकृतायास्तृतीयकरणी भवतीति नवमस्तु भूमेर्भागो भवति । बौ. शु. १.८२.

The meaning is " the altar for the *Pitṛyajña* is to be formed with the third part of the side of the *Mahāvedi*, so that its area will be equal to the ninth part of the *Mahāvedi*". Baudhā. Śl. 1.82.

अष्टात्रिंशत्यूनं पदसहस्रं महावेदिः । आप.शु.पृ. ८३.

" The area of the *Mahāvedi* is 972 sq. *padas*". Āp. Śl. P. 83.

त्रीणि चतुर्विंशानि पदशतानि सौत्रामणिकी वेदिः । आप. शु. पृ. ८६.

(अ) त्रीणि चतुर्विंशाधिकानि पदशतानि वेदितृतीयम् । बौ. शु. १.८६.

" The area of the *Sautrāmaṇī vedi* is ($\frac{1}{3}$rd of *Mahāvedi* = $\frac{1}{3} \times 972 = 324$) 324 sq. *padas*. " The square root of 324 is 18 and therefore, the side of the square of this *vedi* will be 18 *padas* as stated by Baudhāyana and Āpastamba.

महावेदेस्तृतीयेन समचतुरस्रकृताया अष्टादशपदा पार्श्वमानी भवति । बौ. शु. १.८६.

" The side of that area which is formed into a square, with the third part of the *Mahāvedi* is 18 *padas* long ". Baudhā. Śl. 1.86.

The area of the *vedi* for the *Pitṛyajña* will be 1/9 × 972 = 108 sq. *padas*. As it was not very easy for the sacrificer to find out the side of the square during those days it was necessary to find out the same by some other means.

(E) It is also said that the altar for the *Pitṛyajña* should be square shaped. The sides of this square should face the cardinal directions i.e. the corners of the new square will face the main directions, and its area will be equal to one sq. *puruṣa*.

पैतृक्यां द्विपुरुषं समचतुरस्रं कृत्वा करणीमध्ये शङ्कवः स समाधिः। का.शु. २.६.

" In the case of the *Paitṛkī vedi* (to be constructed for the *Mahāpitṛyajña* in the *Śākamedha parvan* of the *Cāturmāsya*, one should construct a square, having an area of two square *puruṣas*. Drive pegs at the centres of all sides. The lines joining these pegs form the square desired. (The area of the new square so formed will be equal to one sq. *puruṣa*) ". Kāt. Sl. 2.6.

यजमानमात्री चतुःस्रक्तिर्भवतीत्येकेषाम् । बौ. शु. १.८३.

"It has the measure of the sacrificer (*puruṣa*) and is four-cornered". Baudhā. Śl. 1.83.

(1) To construct a square having an area of two sq. *puruṣas* requires the help of the theorem of Pythagoras or the theorem of the hypotenuse, as the side of the square will be equal to $\sqrt{2}$.

(2) Moreover, the figure for the new altar formed for the *Pitṛyajña* (having an area of one sq. *puruṣa*) with the help of the sqare having an area of two sq. *puruṣas* leads indirectly to the solution of the general proposition. "The square on the hypotenuse of a square will produce double the area of the square produced by its side."

(F) The *Gārhapatya* mound is circular, that of the *Āhavanīya* is square and that of the *Dakṣiṇāgni* is semi-circular. All these are about one sq. *aratni* in area. Pp 994–Vol II–Part II from the History of Dharma-Śāstra by M.M. Dr. P. V. Kane. Foot Note 2249.

If the area of the circle is to be equal to the area of the semi-circle—then it is quite clear that the diameter of the semi-circle and the diameter of the original circle must be in the ratio of $\sqrt{2} : 1$ and hence one must take the help of the theorem of the Hypotenuse.

IV. CONCLUSION : Thus it will be seen that :

(1) The construction of the *Mahāvedi*, the various halls etc. could be easily produced : (a) with the help of the fixing of the East-West line and the rt. ∠ed △ formed with the help of the various rt. ∠ ed △s formed with the use of the *nirañchana* mark or (b) with the use of the new sets of rt. ∠ed △s, which were not known earlier, But

(2) The construction of the (a) *Aśvamedha* altar, (b) *Sautrāmaṇī*. (c) *Pitṛyajña vedi* of both the shapes and (d) the fire-places of both the *Gārhapatya* and the *Dakṣiṇāgni* was not possible without the theorem of the hypotenuse.

In order to achieve this, they found the following two enunciations and applied them where required. These are :

दीर्घचतुरस्रस्याक्ष्णयारज्जुस्तिर्यङ्मानी पार्श्वमानी च यत्पृथग्भूते कुरुतस्तदुभयं करोतीति क्षेत्रज्ञानम् । का. शु. २.११.

(1) "The diagonal of a rectangle produces by itself both the areas, which the two sides (forming the right angle) produce separately; (i.e. in the case of a rt. ∠ed △ the square on the hypotenuse is equal to the sum of the squares on the two adjacent sides)" Kāt. Śl. 2.11.

समचतुरस्रस्याक्ष्णयारज्जुर्द्विकरणी । का. शु. २.१२.

(2) "In a square, its diagonal produces an area, double that of its side" Kāt. Śl. 2.12.

Thus the development and the manner how it took place regarding the theorem of the Hypotenuse is observed step by step. But this creates two more problems which are worthy of investigation :

These are : To find out the beginning :

(1) When the *Aśvamedha* and other sacrifices originally started, as some of them are dependent on the *Mahāvedi*, it goes without saying that the *Mahāvedi* was there first to start with.

(2) When once the first sacrifice with the basic *citi* was over, the addition of an area of one square *puruṣa* was to be added to every successive *citi*, when the next sacrifice was to be performed, and lastly

(3) The history of the three sacred fires.

If this is neatly traced the date of the discovery of the theorem of the Hypotenuse will still be prior to Baudhāyana Sulba Sūtra i.e. even earlier than 800 B.C. Kapardiswāmī has generalised a method taught by the *Śulbas* for finding the rational rt. ∠ed △s having given a leg. He says :

"The added portion is divided into as many parts as the number obtained by dividing the (given) leg with the added portion by half the added portion, put the *nirañchada* mark by diminishing the added portion by one part.[1]"

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## CHAPTER 2

### History of Maintaining the three Sacred Fires ; and the Foundation laid by them in the Progress of Geometry

Early Indians used to maintain five fires in the house. Out of these, three were maintained for sacred purposes and two were for household.

The three sacred fires are ( i ) *Gārhapatya* ( ii ) *Āhavanīya* and ( iii ) *Dakṣiṇāgni*.

The two household fires are ( i ) *Āvasathya* and ( ii ) *Sabhya*.

The two household fires and their use will be described later. The shapes of the three sacred fires and their use will be described later.

The use of the two household fires was as follows :—

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(1) करविन्दीय व्याख्या पृ. १७ आप. शु. सू.

*Āpastambha śrauta sūtra* ( 4 7-8 ) says

“ अग्रेणाऽऽहवनीयं सभायां सभ्यः । तं पूर्वेणाऽऽवसथ आवसथ्यः ” ॥

Dr. *P. V. Kane* in his book “The history of *Dharmaśāstra*[1]” writes: ‘‘According to *Medhātithi* on *Manu* III, the *Sabhya* fire is that which is kept burning in the hall of a rich man for the removal of cold and for the diffusion of warmth.[1]”

The underlying idea of this emphasis on the worship of the *Agni* seems to be that the oblations offered into the fire reach the sun that sends rain, from which springs corn that is the sustenance of all beings.

The attention of the early Indians was drawn to this problem since the time of *Ṛgveda* ( 3,000 B. C. ).

*Śatapatha Brāhmaṇa* states that the altars for the gods should be squrae in shape, those for demons should be circular.[2] And the *smaśāna-cit* should be square or triangular one as stated by *Satyāṣāḍha śrauta sūtra.*[2]

( A ) The first clear discription of the *Gārhapatya* as a circle of sq. *vyāma* and of the *Āhavanīya* being a square of the same size appears in the *Śata : Brāh* : etc.

It measures a fathom ( *vyāma* ) in diameter...........The *Gārhapatya* is circular.[4]

The altar of *Gārhapatya* is either a square or a circle and measures one sq. *puruṣa.*[5]

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(1) Cp सभ्यो नाम यो महासाधनस्य शीतापनोदार्थमेव बहुषु देशेषु व्यवह्रियते । वृ. गौ. कं. १३. Cf. also आवसथ्यं तु ये चाग्निं पचनाग्निं प्रचक्षते । तेषां सभागतो वह्निः सभ्य इत्यभिधीयते । अग्रेणाहवनीयं सभायां सभ्यः । आप.श्रौ. ४.४.७.

(2) चतुःस्रक्ति देवाश्च........ । परिमण्डलानि असुराश्च....। शत.ब्रा. १३.८.५

(3) स्मशानचितं चिन्वीत यः कामयेत पितृलोक ऋध्नुयामिति चतुरस्रः परिमण्डलो वा । (यथा महापैतृकी वेदिरित्येकेषाम् ) । त्र्यश्रिरित्येकेषाम् । सत्याषाढ श्रौ.सू.१२.८.१५

(अ) द्रोणचितं चिन्वीतान्नकाम इति द्वयानि तु खलु द्रोणानि चतुरस्राणि परिमण्डलानि च तत्र यथाकामी । सत्या. श्रौ. सू. १२.८.९.

(4) व्याममात्री भवति । ...परिमण्डला भवति । ... गार्हपत्यः परिमण्डलम् । शत. ब्रा. ७.१.१.३७.

(5) गार्हपत्यचितेरायतनं व्यायायमात्रं चतुरस्रं परिमण्डलं वा । आप.श्रौ.सू.१६.१४.१.

On the East an altar having an area of one sq. *puruṣa* is constructed. This is circular.[1]

On this point (in Vol. XLIII on page 307 in note 1 of the Sacred books of the East) the following note appears :

“ The (original square) *Āhavanīya* is laid down, but this ultimately makes way for the new circular *Gārhapatya* hearth built of bricks and having the whole of this fathom for its diameter ”.

(B) *Cinnaswāmī* in his *Yajña-tattva-prakāśa* states as under :

गार्हपत्याग्नेरायतनं वृत्ताकारम् । आहवनीयस्य चतुरश्रम् । दक्षिणाग्नेरर्धचन्द्राकृति । गार्हपत्यायतनमध्यात् प्राच्यां दिशि षण्णवत्यङ्गुलमितायां भूमावाहवनीयमध्यं स्यात् ।........ गार्हपत्यायतनस्य आग्नेय्यां दिशि तत्संलग्नप्रायमेव दक्षिणाग्नेरायतनं भवति ।

(यज्ञतत्त्वप्रकाश, चिन्नस्वामी )

“The *Gārhapatya* altar is circular in shape; the altar for *Āhavanīya* is a square and that of *Dakṣiṇāgni* a semi circle. The centre of the *Āhavanīya* altar is 96 *aṅgulas* away and is on the East, from the centre of the *Gārhapatya* altar... *Dakṣiṇāgni* is situated on the South-East of *Gārhapatya.*”

This is corroborated as under :

‘‘The *Āhavanīya* fire place is a square, *Gārhapatya* is round and *Dakṣiṇāgni* is in the form of half the orb of the Moon”. Vide Vṛddha Gautama p.604 by Jīvānanda), The above statement is from the History of *Dharmaśāstra* by Dr. P. V. Kane, Vol. II, part I, p. 679, F. Note 1618.

He further adds the following :

“ The *Gārhapatya* mound is circular, that of the *Āhavanīya* is square and that of the *Dakṣiṇāgni* is semi-circular. **All the three are about one sq.** *aratni* in **area**”. (Page 994 of Vol. II–part II of *Dharma śāstra* by M. M, Dr. P.V. Kane). The above quotation is most important in as much as it clearly states that the **areas of all the three altars** having different shapes namely : a square; a circle and a semi-circle; **must be equal.**

The source of this quotation is not given nor I have been able to trace it but almost all the known *śulba sūtras* give the dimensions for the various shapes and the areas of those are nearly equal.

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(1) अपरेणाहवनीयं यजमानमात्रीं वेदिं करोति । सत्या. श्रौ. सू. ३.३.

See the statement given below :

| *Śulba sūtra* | *Gārhapatya* (a circle) Radius (*aṅgulas*) | Area sq. *aṅg.* | *Āhavanīya* (a square) Side (*aṅg.*) | Area sq. *aṅg* | *Dakṣiṇāgni* (a semi circle) Dia. *aṅg.* | Area sq. *aṅg.* |
|---|---|---|---|---|---|---|
| (1) *Baudhāyana* | 12 | 452·38 | $21\frac{9}{34}$ | 452·18 | $34\frac{5}{17}$ | 461·8 |
| (2) *Āpastamba* | — | — | 24 | 576 | — | — |
| (3) *Kātyāyana* | 13½ | 572·56 | 24 | 576 | | |
| (4) ,,<br>A | 14 | 616.73 | 24 | 576 | 32 or 38 | 402·12<br>567·06 |
| (5) *Mānava* 14 | 1 yava | 628.30 | 24 | 576 | 19½ | 597 |

(1) Measurements are given by the *Baudhāyana's* Commentator.
(2) Measurements are given by Commentator *Karavindaśwamī*.
(3) Measurements are given by Commentator *Vidyādhara Śarmā*.
(4) Measurements are given Chap. VII—Verses 37 and 38 of Kāt. Śl.
(5) Measurements are given by N, K. Muzumdar.

All the above areas are based on the value of $\pi = 3{\cdot}14159$. The mention of this *Gārhapatya* fire is found in the *Ṛgveda* also.

On this point B. B. Dutt in his " The science of *Śulba* " says : " Hence it seems that the problem of : (1) The squaring of the circle and (2) The Theorem of the square of the hypotenuse (at least in its simplest form) are as old in India, as the time of *Ṛgveda*.

Burk (the translator of the *Āpastamba śulla sūtra* in 1902) says : " I shall only emphasize the fact that the Indians must have understood, really in the time of the *Taittirīya Saṁhitā*, how to solve the problem of the circling of the square (although on very primitive methods) ".

The place occupied by these three sacred fires, in the case of the *Mahāvedi* was in the hall called *Prācina-vaṁśa*.

Similar instances of constructing a square, first of 7½ sq. *puruṣas* in area and then transforming the same into a circle are there. *Dhiṣṇya* may be a squre or a circle : *smaśāna citi* whose area is of one sq. *puruṣa* may be in the form of a square or a circle.

Thus to find the area of a circle, a square and a semi-circle was necessary and that too equal to one another.

**This Very fact laid the foundation of Geometry in India.**

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## CHAPTER 3

### Squaring the Circle or Circling the Square

All the *sulba sūtras* deal with this problem. The problem is to provide a square, equal in area to a circle or to draw a circle equal in area to a square.

This problem has attracted the attention of the greatest mathematicians of the world and it is still eluding them. *Rāmānujan*, the greatest Indian mathematician of the present century also tried to solve it.

It should be noted, that on the circle, the Greek mathematicians of this time had done very little work of any importance.

As early as in the time of the Greek mathematicians circle squaring occupied the attention of non-mathematicians.

In the year 1775, the Paris Academy found it necessary to protect its officials against the waste of time and energy involved in examining the efforts of circle squares. It passed a resolution recorded in the minutes of the Academy, that no more solutions were to be examined of the problems of (1)The duplication of the cube (2) the trisection of the angle and (3) the quadrature of the circle, etc.

In the earlier part, this problem was set back in the West, due to the backward condition of Arithmetic owing to the fact that the present system of numerical notation, which was invented in India, was not known to the Western world.

This problem necessitates the knowledge of the following : (a) Area of a circle. (b) Area of a square. (c) Relation between the circumference and the diameter of a circle, which is known as $\pi$ and (d) The value of irrational or the surd viz. : $\surd 2$.

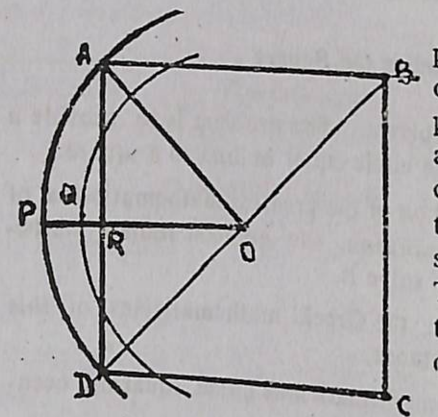
**Ist Problem : To turn a square into a circle.**

(१) चतुरस्रं मण्डलं चिकीर्षन्मध्यादंसे निपात्य पार्श्वतः परिलिख्य तत्र यदतिरिक्त भवति, तस्य तृतीयेन सह परिलिखेत् स समाधिः। का.शु.३.१३.

(अ) चतुरस्रं मण्डलं चिकीर्षन् मध्यात् कोट्यां निपातयेत् । पार्श्वतः परिकृष्यातिशयतृतीयेन सह मण्डलं परिलिखेत् । साऽनित्या मण्डलम् । यावत् हीयते तावदागन्तु । आप. शु. पृ. ४९.५०.

(ब) चतुरस्रं मण्डलं चिकीर्षन्नक्ष्णयार्धं मध्यात् प्राचीमभ्यापातयेद्यदतिशिष्यते तस्य सह तृतीयेन मण्डलं परिलिखेत् । बौ. शु. १.५८.

" If you desire to transform a square into a circle, draw half the diagonal from the centre towards a corner (*aṁsa*), then with this as radius draw a circle. Then with ⅓rd part of the radius, that lies outside the square, describe another circle. This circle will be equal in area to that of the square ". Kāt. Śl. 3.13.



Let ABCD be a square. It is proposed to draw a circle equal in area to that of this square. With radius OA draw a part of the circle APD. Bisect AD at R and then extend OR to P, a point on the circumference of the circle. Divide RP the part of the radius, which is lying out side the square into three equal parts. Then with the third part of this RP, added to half the side of the square as radius describe a circle.

Fig 36

This circle (as per above rule) should be equal in area to that of a square.

Let 2a be the side of the given square, and r = the radius of the circle, which will be equal in area to the given square.

AD = 2a. QR = $\frac{1}{3}$ PR. ∴ AR = RD = a and OQ = r.

∴ $OA^2 = OR^2 + AR^2 = a^2 + a^2 = 2 \cdot a^2$ ∴ $OA = \sqrt{2} \cdot a$.

Now . $OP = OA = \sqrt{2} \cdot a$ and $OQ = r$.

$PR = OP - OR = \sqrt{2} \cdot a - a = a(\sqrt{2} - 1)$.

$QR = \frac{1}{3} PR = \frac{1}{3} \cdot a(\sqrt{2} - 1)$

$OQ = OR + RQ = r = a + \frac{1}{3} a(\sqrt{2} - 1)$.

$$\therefore \quad r = a + \frac{a}{3}(\sqrt{2} - 1) \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

The value of r as arrived at in (1) above cannot be finally determined, unless and until the value of √2, an irrational quantity, is found out.

How the value of √2 was derived, by the ancient Hindus is explained in another article. The value obtained is

$$\sqrt{2} = 1{\cdot}4142156 \quad \ldots \quad \ldots \quad (2)$$

$$\text{Now } r = a + \frac{a}{3}(\sqrt{2} - 1) \quad \ldots \quad \ldots \quad (1)$$

Substituting the value of √2 in (1) above we have

$$r = a + \frac{a}{3}(1{\cdot}4142156 - 1)$$

$$= a + \frac{a}{3}(0{\cdot}4142156) = a + a \times 0{\cdot}13807186.$$

$$= 1{\cdot}138a.$$

$$\text{Area of the circle} = \pi r^2 = \frac{22}{7} \times (1{\cdot}138a)^2$$

$$= 3{\cdot}14159 \times 1{\cdot}29a^2$$

$$= 4{\cdot}0526511a^2 \quad \ldots \quad (3)$$

$$\text{Area of the square} = 4a^2 \quad \ldots \quad (4)$$

∴ The area of the circle is greater by 0·0526511.

This is more by 1·25%. Here both the values of π = 3·14159 and the value of √2 = 1·4142155 are taken correct upto five places of decimals, in arriving at the area of the circle as desired above.

Let us try to find out the value of π.

$$\because \text{ The area of the circle} = \pi r^2 = \pi a^2 \times 1{\cdot}138^2$$

$$= 4a^2 = \text{Area of the square.}$$

$$\therefore \quad \pi = \frac{4a^2}{a^2 \times 1{\cdot}138^2} = \frac{4}{1{\cdot}295} = 3{\cdot}08 \quad \ldots \quad (5)$$

In the *Āpastamba śulba sūtra* (pp. 49–50) the words *Sā'nityā maṇḍalam* which are already there, were omitted while discussing the above. Similarly, the commentator on *Baudhāyana* adds the words to his *sūtra*, *Sā'nityā Maṇḍalam.* Similar statements are made while transforming a circle into a square. I will deal with these statements later.

**2nd problem : To transform a circle into a square.**

(A) मण्डलं चतुरस्रं चिकीर्षन् विष्कम्भं पञ्चदश भागान् कृत्वा द्वावुद्धरेच्छेषः करणी ।

का. शु. ३.१४.

मण्डलं चतुरस्रं चिकीर्षन् त्रिष्कम्भं पञ्चदश भागान् कृत्वा द्वावुद्धरेत् ।

त्रयोदशावशिष्यन्ते । साऽनित्या चतुरस्रम् । आप. शु. पृ. ५१.

मण्डलं चतुरस्रं चिकीर्षन् । अपि वा पञ्चदश भागान् कृत्वा द्वावुद्धरेत् सैषाऽनित्या

चतुरस्रकरणी । बौ. शु. १.६०.

"One desirous of squaring a circle, should divide the diameter of the circle into 15 parts. Remove two parts. the remaining portion (18 parts) will form the gross value of a side of the equivalent square". Kât. Śl. 3.14.

If $2a$ = The side of the square

and $d$ = diameter of the circle, Then

as per above rule, $2a = d - \frac{2}{15}d$ or $a = r - \frac{2}{15}r$.

$$\therefore \quad a = \frac{13}{15} r. \qquad \therefore \quad a = 0{\cdot}87978\, r.$$

As $4a^2 = \pi r^2$.

Substituting the values of $a$ and $r$ arrived at as above.

We have $4 \times (0.87978)^2 \cdot r^2 = \pi r^2$.

$$\therefore\ \pi = 4 \times 0{\cdot}7740128484.$$
$$= 3{\cdot}09. \qquad (6)$$

(B) **A second method for the above is suggested by *Baudhāyana* only, which is as follows :**

मण्डलं चतुरस्रं चिकीर्षन् विष्कम्भमष्टौ भागान् कृत्वा भागमेकोनत्रिंशधा विभज्याष्टाविंशतिभागानुद्धरेद् भागस्य च षष्ठमष्टमभागोनम् । बौ. शु. १.५९.

"If you wish to transform a circle into a square, divide its diameter into 8 parts and then divide one of these parts into 29 parts and subtract 28 of these and also the sixth part (of the preceding sub-division) less the eighth part (of the last)." Baudh. Śl. 1.59.

$$\therefore\ 2a = \frac{7d}{8} + \frac{d}{8} - \frac{28}{8{\cdot}29} \cdot d + \left(\frac{8}{8{\cdot}29{\cdot}6} - \frac{8}{8{\cdot}29.6.8}\right) d.$$
$$\therefore\ 2a = d - \frac{d}{8} + \frac{d}{8{\cdot}29} - \frac{d}{8{\cdot}29}\left(\frac{1}{6} - \frac{1}{6\,8}\right)$$

Since $d = 2r$, where $r$ is the radius of the circle.

$$a = r - \frac{r}{8} + \frac{r}{8{\cdot}29} - \frac{r}{8{\cdot}29}\left(\frac{1}{6} - \frac{1}{6{\cdot}8}\right)$$
$$= r\left(1 - \frac{1}{8} + \frac{1}{8{\cdot}29} - \frac{1}{8{\cdot}29{\cdot}6} + \frac{1}{6{\cdot}8{\cdot}29{\cdot}8}\right)$$
$$= r\,(1 - 0{\cdot}125 + 0{\cdot}004 - 0{\cdot}0007 + 0{\cdot}00008)$$
$$\therefore\quad a = r\,(1 - 0{\cdot}12022) = 0.87978r.$$

Now the area of the square = area of the circle.

$$\therefore\ 4a^2 = \pi r^2.$$

Substituting the value of $a$ in the above.

$$\therefore\quad 4 \times (0\,87978r)^2 = \pi r^2. \quad \therefore\ \pi = 4 \times (0.87978)^2 = 4 \times 0{\cdot}774,$$
$$= 3{\cdot}09. \qquad (7)$$

If however, "the diameter is divided into 13 parts (instead of 15 as stated above) the result will be still nearer[1]". The *Mānava śulba* also gives the same value,

$$\therefore\ 2a = d - \frac{1}{9} d \quad \text{or} \quad a = r - \frac{1}{9} r$$
$$\therefore\ a = \frac{8}{9} r, \qquad \therefore\ a = 0{\cdot}888\, r.$$

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(1) इदं स्थूलमानेनोक्तम् । सूक्ष्मं तु अष्टादशभागान् कृत्वा द्वौ त्यजेत् ।
तदुक्तं वार्तिके–'वृत्तव्यासं नवांशे वा परिहृत्याथ तां वदेत् ।
करणी चतुरस्रार्थमल्पमेवान्तरं भवेत्' ॥ इति ॥ का. शु. सू. महीधरभाष्य पृ. २२.

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Since $4a^2 = \pi r^2$

$$\therefore\ 4 \times (0{\cdot}88)^2 \cdot r^2 = \pi r^2. \quad \therefore\ \pi = 4 \times (0\,88)^2 = 3{\cdot}1604 \quad ..\ (8)$$

When *Dvārkānāth Yajvā*, a great commentator on *śulba* found that the value arrived at from the different solutions given above by the *śulbakāras* was approximate, he tried to improve the same. He ultimately found the value of $\pi$ as follows : $\pi = 3{\cdot}157991$.

The values of $\pi$ as given by both *Mānava* ($\pi = 3{\cdot}1604$) and *Dvārakānāth Yajvā* ($\pi = 3{\cdot}157991$) are far better and nearer to the value of $\pi = 3{\cdot}14159$ as adopted at present. The value of $\pi = 3{\cdot}09$ was arrived at by the *Śulbakāras*.

The value of $\pi$ was however, still being improved.
*Āryabhaṭṭa* in his *Āryabhaṭṭīya* gives this value as :—

"चतुरधिकं शतमष्टगुणं द्वाषष्टिस्तथा सहस्राणाम् ।
अयुतद्वयविष्कम्भस्यासन्नो वृत्तपरिणाहः ॥ आर. शु. सू. पृ. ५३.

It states $\pi = \dfrac{62832}{20000} = 3{\cdot}1416$. This value is correct upto 4 places of decimals. **And still he knew that it was approximate.**

E. W. Hobson in his book on "squaring the circle" published by the Cambridge University Press in 1913 states that the values of $\pi$, even though these were found to be quite different from and greater than 3, the value of $\pi = 3$ was current, for many centuries, practically all over the world. The values of $\pi$ derived from the *śulba sūtras* range from 3·09 to 3·16.

II. In some of the statements made in the *sūtras* above, while transforming a square into a circle or a circle into a square, the commentators on *Baudhāyana* and *Āpastamba* add the words *sā nityā* which means that the thing is exact or *Sā'nityā* is added to the rule which means that the thing is approximate.

This meaning of these two words used depends on how the words are read or written—' if read jointly their meaning will be 'approximate', if read separately, they will mean 'exact'; in my opinion, the words should clearly be taken to mean as "approximate".

The reasons are quite clear :

The most important point to be discussed in this regard is (a) whether the area of the square or the area of the circle as determined by the general rules quoted above was exact or approximate.

Dr. G. Thibaut thinks that the values obtained are no doubt approximate but from the way they appear in the text, their meaning in one case must be taken as 'exact' according to *Āpastamba* and 'approximate' as per *Baudhāyana's* Commentator, in the case of constructing a circle equal in area to that of a square.

While in the case of constructing a square equal in area to that of a circle, both *Baudhāyana* and *Āpastamba* agree that the result obtained as per rule is only 'approximate'.

Let us now turn our attention to what Mr. T. L. Heath has to say in this respect. He says this while discussing the value of $\sqrt{2}$, which is less by $\frac{1}{1156}$ according to Dr. Thibaut. Mr. T. L. Heath further observes : "He was leaving out of account the small fraction $\frac{1}{1155}$. However, as the object was purely practical he would without hesitation ignore this as being of no practical importance and therefore, the formula would be handed down and taken as a matter of course without arousing suspicion as to its accuracy. This supposition is confirmed by reference to the sort of rules, which the Indians allowed themselves to regard as accurate. Thus *Āpastamba* himself gives a construction for a circle equal in area to a given square, which is equivalent to taking $\pi = 3.09$ and yet observes that it gives the required circle 'exactly'. While his construction of a square equal to a circle which he equally calls 'exact' makes the side of a square equal to $\frac{13}{15}$ths of the diameter of a circle.

There are two problems before the *Sūtrakāras*. The problems with their solutions are identically stated by all of them with the following exceptions :

| | A circle equal to a square | A square equal to a circle |
|---|---|---|
| *Baudhāyana* | Approximate | * |
| *Āpastamba* | Approximate | Exact |

* Commentator states that it is not exact.

*Āpastamba* came into the field after *Baudhāyana*.

He follows in his footsteps and quotes the same as 'approximate'. But dealing with the 2nd problem *Āpastamba* adds the words *Sā nitā* which mean exact. If the meaning is taken to mean as stated above what induced *Āpastamba* to make such a statement is not known. This means that there was no improvement in the solution at all. But *Mānava* improved the solution of the same problem by taking the side of the square equal to 8/9 th of the diameter of the circle.

If the solution was exact according to *Āpastamba* why should *Mānava*, writing on the same subject of *śulba*, come forward to improve it ? The answer is simple. It is this that *Āpastamba* never thought of saying that the solution of the rule was 'exact', but he thought clearly that the solution as found out was only approximate. If such was the state then where lies the mistake ?

Those who are conversant in reading the manuscripts written in the Sanskrit language know that the sentences after sentences in the manuscripts are written continuously without writing the words separately as is done at present while writing and printing. It is quite evident therefore, that the first writer of

the *Āpastamba śulba sūtra* while copying from the manuscript must have written the words *Sā nityā* separately instead of writing them *Sānityā* jointly as one word and hence the mistake in translation.

In fine; it must be admitted, with whatever limited means at their disposal, the early Hindus, say about 3,000 years back, were able to find out the following :

(1) Area of the circle equal in area to a square approximately,

(2) Area of a square, approximately equal in area to a circle.

(3) The relation between $\frac{\text{circumference}}{\text{diameter}}$. now known as $\pi$.

(4) Ths value of the irrational or surd viz. $\sqrt{2}$.

Re : The last item, please see article 'on the value of $\sqrt{2}$',written separately

III. *Baudhāyana* has laid down a rule for transforming a circle into a square. This rule is not stated by both *Āpastamba* or *Kātyāyana*. The rule wil be restated here and an attempt to show as to how the rule must have been arrived at is given below in the words of Shri B. B. Dutt and Prof. L. V, Gurjar. The rule is :

"If you wish to square a circle, divide its diameter into 8 parts. Then divide one part into 29 parts and leave out 28 of these; and also the sixth part (of the preceding sub-division) less the eighth part (of the last)".

Thus if 2 a denotes the side of a square, equal in area to a circle of diameter $d$ then :

$$2a = \frac{7}{8}d + \left(\frac{d}{8} - \frac{28d}{8.29}\right) + \left(\frac{d}{8.29.6} - \frac{d}{8 \cdot 29 \cdot 6 \cdot 8}\right)$$

$$\text{or} \quad 2a = d - \frac{d}{8} + \frac{d}{8 \cdot 29} - \frac{d}{8 \cdot 29}\left(\frac{1}{6} - \frac{1}{6.8}\right)$$

Since $d = 2r$ where $r$ is the radius of the circle.

$$a = r - \frac{r}{8} + \frac{r}{8 \cdot 29} - \frac{r}{8.29.6} + \frac{r}{8.29.6\ 8}$$

This result was probably obtained from the previous one by inversion.

$$r = \frac{a}{3}(2 + \sqrt{2})$$

$$\therefore \quad 2a = \frac{3}{2 + \sqrt{2}} \cdot d$$

Substituting the value of $\sqrt{2}$ viz, $\frac{577}{408}$ we have

$$2a = \frac{1224}{1393} \cdot d$$

(A) Prof. Thibaut tries to give an explanation that is based upon the work of commentators of a later date (Asiatic Journal of Bengal 1875) [1].

*Baudhāyana* assumed a = 24 *aṅgulas*.

$$\therefore \frac{a}{2} = 12 \text{ } \textit{aṅgulas} = 12 \times 34 = 408 \text{ } \textit{tilas}.$$

Now if a is the side of a square, its diagonal will be = $\sqrt{2}\cdot a$.

$$\therefore \tfrac{1}{2} \text{ the diagonal} = \frac{\sqrt{2}\cdot a}{2} = \left(\frac{a}{2}\cdot\sqrt{2}\right) \text{ or } \frac{a}{\sqrt{2}}$$

$$\therefore \frac{a}{2}\sqrt{2} = 12\left(1+\frac{1}{3}+\frac{1}{3.4}-\frac{1}{3.4.34}\right) = 16\ \frac{33}{34}$$

$$= 16 \text{ } \textit{aṅgulas} \text{ and } 33 \text{ } \textit{tilas}.$$

$$\therefore \frac{a}{\sqrt{2}}-\frac{a}{2} = 16\frac{33}{34} - 12 = 4 \text{ } \textit{aṅgulas} \text{ and } 33 \text{ } \textit{tilas} = 169 \text{ } \textit{tilas}.$$

$$\therefore \frac{1}{3}\left(\frac{a}{\sqrt{2}}-\frac{a}{2}\right) = \frac{1}{3}\cdot 169 = 56\frac{1}{3} \text{ } \textit{tilas},$$

The radius of the circle a $\frac{1}{3}\left(\frac{a}{\sqrt{2}}-\frac{a}{2}\right) = 408 + 56\frac{1}{3} = 464\frac{1}{3}$ *tilas*

In other words if ½ the side of the square is 408 *tilas* long, the length of the radius of the circle is 464⅓ *tilas*. By avoiding fraction, the radius of the circle will be 1393 and half the side = 1224 *tilas*.

⅛th of 1393 = 174⅛; this multiplied by 7 = 1218⅞

Difference between $1224 - 1218\frac{7}{8} = 5\frac{1}{8}$

Dividing 174 (*Baudhāyana* takes 174 instead of 174 1/8, neglecting the fraction as either insignificant or more likely as inconvenient) by 29; we get 6; subtracting from 6 its sixth part we get 5 and adding to this 1/8th of 1/6th of 6 we get 51/8. In other words:

$$1224 = \frac{7}{8}+\frac{1}{8\cdot 29}-\frac{1}{8\cdot 29\cdot 6}+\frac{1}{8\cdot 29\cdot 6\cdot 8} \text{ of } 1393,$$

(due allowance made for the neglected 1/8).

(B) In the opinion of Cantor the series was probably obtained thus:

$$\frac{1224}{1393} = \frac{7}{8}+\frac{1}{8\cdot 29}-\frac{1}{8\cdot 29\cdot 6}+\frac{1}{8\cdot 29\cdot 6\cdot 8}-\frac{41}{8\cdot 29\cdot 6\cdot 8\cdot 1393}$$

The last term is nearly 1/34th of the term preceding it and so may be neglected as being comparatively small. Hence we get:

$$2a = \frac{7}{8}d + \frac{1}{8\ 29}\cdot d - \frac{1}{8\cdot 29\cdot 6}\cdot d + \frac{1}{8\cdot 29\cdot 6\cdot 8}\cdot d$$

---

[1] Both Kapardisvāmī and Karawindaswāmī have suggested this method. Apa. Śl. p. 27 and p. 31.

(C) Muller conjectures the procedure adopted to have been as follows:

$$2a = \frac{3}{2+\sqrt{2}}\cdot d = \frac{3}{2}\cdot\frac{\sqrt{2}}{1+\sqrt{2}}\cdot d.$$

$$= \frac{3}{2}\times\frac{17-\frac{1}{34}}{29-\frac{1}{34}} = \frac{51-\frac{3}{34}}{58-\frac{2}{34}}$$

$$\text{Since } \sqrt{2} = \frac{17}{12}-\frac{1}{12\cdot 34}$$

$$\therefore \frac{51-\frac{3}{34}}{58-\frac{2}{34}} = 1-\frac{7+\frac{1}{34}}{58-\frac{2}{34}}$$

$$\therefore \frac{7+\frac{1}{34}}{58-\frac{2}{34}} = \frac{1}{8}\times\frac{56\frac{8}{34}}{58-\frac{2}{34}} = \frac{1}{8}\left(1-\frac{2-\frac{10}{34}}{58-\frac{2}{34}}\right)$$

$$\therefore \frac{2-\frac{10}{34}}{58-\frac{2}{34}} = \frac{1}{29}\times\frac{2-\frac{10}{34}}{2-\frac{2}{34\cdot 29}} = \frac{1}{29}\left(1-\frac{\frac{10}{34}-\frac{2}{34\cdot 29}}{2-\frac{2}{34\ 29}}\right)$$

$$\therefore \frac{\frac{10}{34}-\frac{2}{34\cdot 29}}{2-\frac{2}{34\cdot 29}} = (\frac{5-\frac{1}{29}}{34-\frac{1}{29}} = \frac{1}{6}\times\frac{30-\frac{6}{29}}{34-\frac{1}{29}} = \frac{1}{6}\times\frac{4-\frac{5}{29}}{34-\frac{1}{9}}.$$

$$\therefore \frac{4+\frac{5}{29}}{34-\frac{1}{29}} = \frac{1}{8}\times\frac{32+\frac{40}{29}}{34-\frac{1}{29}} = \frac{1}{8}\left(1-\frac{2-\frac{41}{29}}{34-\frac{1}{29}}\right)$$

Thus we have:

$$\frac{3}{2+\sqrt{2}} = 1-\frac{1}{8}+\frac{1}{8\ 29}-\frac{1}{8\ 29}\left(\frac{1}{6}-\frac{1}{6\cdot 8}\right)-\frac{1}{8\cdot 29\cdot 6\cdot 8}\times\frac{2-\frac{41}{29}}{34-\frac{1}{29}}$$

The last term may be neglected as being too small.

$$\because\ 2a = d - \frac{d}{8} + \frac{d}{8\cdot29} - \frac{d}{8\cdot29}\left(\frac{1}{6} - \frac{1}{6\cdot8}\right)$$

$$= \frac{7}{8}d + \frac{d}{8\cdot29} - \frac{d}{8\cdot29\,6} + \frac{d}{8\cdot29\,6\,8}$$

(D) Now let us see what Prof Gurjar has to suggest about this procedure. He says :

To my mind it appears that this is not the method by which *Baudhāyana* might have arrived at such a wonderfully accurate result in consideration of the same value of $\pi$ in both the cases.

The logical interpretation must have been as follows :

There is no need to take a square, whose side is 24 *aṅgulas*.

For $d = \frac{a}{3}(2 + \sqrt{2})$

$$\therefore\ \frac{d}{a} = \frac{1}{3}\left(2 + 1 + \frac{1}{3} + \frac{1}{3\,4} = \frac{1}{3.4.34}\right) = \frac{1393}{1224}$$

$$\therefore\ \frac{a}{d} = \frac{1224}{1393}$$

To generalise this rule it was essential to express 1224 in terms of 1393. *Baudhāyana* then found the greatest possible integral number that would divide both the numerator and the denominator giving small remainders. To obtain this number divide 1393 by 1224 and the remainder is 169 the possible common measure for the two numbers 1393 and 1224. If 1393 is divided by 169, the quotient is 8 and the remainder is 41. While 1224 is divided by 169, the quotient is 7 and the remainder is 91. In the first case, to reduce the remainder to its lowest value by keeping the quotient 8 and in the second case to reduce the remainder to its lowest value by keeping the quotient 7, the greatest common measure would be 174.

Since 174 × 8 = 1392 and the remainder left is 1 and 174 × 7 = 1218 and the remainder left is 6. Thus after 169, *Baudhāyana* must have by turn divided both the numbers by 170, 171, 172, 173 and 174 and stopped at 174, since it served the required purpose. This process involves simlpe division and therefore it is easily suggestible. Thus *Baudhāyana* obtained the maximum divisor for minimum remainders.

Since the idea was simply to express 1224 in terms of 1393. Thus :

$$\frac{a}{d} = \frac{1224}{1393} = \frac{\left(\frac{1224}{174}\right)}{\left(\frac{1393}{174}\right)} = \frac{\left(7 + \frac{6}{174}\right)}{\left(8 + \frac{1}{174}\right)} = \frac{7 + \frac{1}{29}}{8 + \frac{1}{6\,29}} = \frac{\frac{7}{8} + \frac{1}{8\,29}}{1 + \frac{1}{6\cdot8\cdot29}}$$

---

$$= \left(\frac{7}{8} + \frac{1}{8\cdot29}\right) \times \left(\frac{1}{1 + \frac{1}{6\cdot8\cdot29}}\right) = \left(1 - \frac{1}{8} + \frac{1}{8\cdot29}\right) \times \frac{1}{\left(1 + \frac{1}{6\cdot8\,29}\right)}$$

Now the value of the factor $\left(\frac{1}{1 + \frac{1}{6\cdot8\cdot29}}\right)$ is obtained by simple division as in.

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 \ \ldots$$

$$\therefore\ \frac{1}{1 + \frac{1}{6\cdot8\cdot29}} = 1 - \frac{1}{6\cdot8\,29} + \left(\frac{1}{6\cdot8\,29}\right)^2 - \left(\frac{1}{6\,8\cdot29}\right)^3$$

$$\therefore\ \frac{a}{d} = \left(1 - \frac{1}{8} + \frac{1}{8\cdot29}\right)\left(1 - \frac{1}{6\,8\cdot29} + \ldots\right)$$

higher powers in the second bracket being neglected as they are small.

So it would be observed that by making use of simple division, the *Sūtrakaras* made use of the Binomial Theorem with the -ve (negative) unit index, and they were perfectly jusitified in doing so as the argument $x = \frac{1}{6\,8.2}$ is less than unity. Throughout, this process is based upon ordinary division and as such it is easily suggestible.

Thus we saw four different statements from four different scholars regarding the procedure that must have been adopted by *Baudhāyana* before stating the rule he framed.

Now, before concluding this article, I would like to place on the record two more statements by Dr. Thibaut on the same subject : These are : (1) While discussing the problem of transforming the circle into a square he says : "But I am afraid we should not be justified in giving to *Āpastamba* the benefit of this explanation. The words "*Yāvudd hīyate*" seem to indicate that he was perfectly satisfied with the accuracy of his method and not superior in this point to so many circle squarers of later times".

Modern circle squarers, well knew the values of both $\pi$ and $\sqrt{2}$. Their problem was only to equate the area of the circle with that of the square. But such was not the case when the rules were being framed by the *Sūtrakāras*.

(2) Regarding the 2nd rule for finding the side of the square the area of which will be equal to that of a circle. Dr. Thibaut says : " This rule for turning a circle into a square is the reverse of the preceding rule for turning a square into a circle. It is based on the remarkably accurate numerical expression for the relation of the diagonal of a square to the side of a square". He further adds : "According to the custom of the class of writings called *sūtras* the method by which the result was obtained is not stated".

Thus, the length of the side, obtained with the help of the above expression of *Baudhāyana* will be more accurate, of a square, the area of which will be equal to that of a circle. Since the value of √2 is approximate, the value of the side of the square based on the value of √2 will also be approximate.

It took more than a century and a half to establish the fact that the values of √2 and π are incommensurable and the same cannot be exactly determined. This was when all sorts of facilities were at hand.

Now 3000 years back, when the *śulba sūtras* were prepared by *Baudhāyana* and others, they noticed that it was most essential to determine the value of the ratio of the circumference and the diameter of the circle now known as π, as well as the value of the diagonal of a square (√2 ) if it be necessary to transform a circle into a square having the same area.

To this end in view, how far they succeeded in their attempt is now thoroughly understood. It is no use, therefore, to belittle their efforts on flimsy grounds. On the other hand one must appreciate their efforts and give credit which they really deserve.

The following information is given by Shri B. B. Dutta.

The value of π quoted by Aryabhatta I, which was π = 3·1416, given in Āryabhaṭṭīya is already quoted.

*Dvārakānath* must be posterior to *Āryabhaṭṭa* (499 A.D.) whom he quotes. He proves with the help of illustrative examples that the methods taught in the *Śulbas* for the squaring of the circle and vice versa do not lead to an accurate result as compared with that obtained by the method of *Āryabhaṭṭa*.

If 2 a be the side of the square equivalent to a circle of radius *r*, then according to the *śulba*.

$$r = a + \frac{a}{3}(\sqrt{2}-1) \quad - \quad - \qquad (1)$$

$$a = r - \frac{r}{8} + \frac{r}{8\,29} - \frac{r}{8\cdot29\cdot6} + \frac{r}{8\cdot29\cdot6\cdot8} \quad - \qquad (2)$$

These lead to π = 3.0883 – ..3·0885 respectively.

*Dvārakānath Yajvan* amends them to :—

$$r = \left(a + \frac{a}{3}(\sqrt{2}-1)\right)\left(1 - \frac{1}{118}\right)$$

$$a = \left(r - \frac{r}{8} + \frac{r}{8\,29} - \frac{r}{8\cdot29\cdot6} + \frac{r}{8\cdot29\cdot6\cdot8}\right)\left(1 + \tfrac{1}{2}\cdot\frac{3}{133}\right)$$

These will work out to π = 3·141109 or 3·157991.

Thus it will be seen that attempts were being made to find out the accurate (approximate) value of π in later times based on same results.

Extract from the " History of Philosophy : Eastern and Western " (Article on Geometry by A. N. Singh pages 434, 435 .

Value of π : Although the Greeks were great geometers they did not succeed in finding a good value of π. They were content with the value of $\pi = \frac{22}{7}$. The Indians however, acquired better approximation to the value of π and as early as A. D. 499. *Āryabhaṭṭa* gave the value.

$$\pi = \frac{62832}{20{,}000} = 3\cdot1416.$$

The values $\frac{22}{7}$ and $\frac{355}{113}$ have been used by Indian Mathematicians but were not popular. The value of π = √10, because of facility in its use, was preferred by some.

The value of π = 355/113 has been used in the *Dhavala*. The Chinese used this value probably under Indian influence. Later Indian works gave values of π correct to nine or more decimal places. In the beginning the Indian mathematicians found better approximations to the value of π by increasing the number of sides of an inscribed regular polygon. Later on in the 16th and 17th centuries they used infinite series for the purpose. Both the above methods were used by European mathematicians but certainly much later than they were used in India.

## CHAPTER IV

### Fixing the Position of the *Dakṣiṇāgni* or a crude attempt to find the Value of √2 and √5

The three sacred fires are : (1) *Gārhapatya*. (2) *Āhavanīya* and (3) *Dakṣiṇāgni*.

The position of the *Dakṣiṇāgni* is described in three different ways by *Baudhāyana* and in two different ways by both *Āpastamba* and *Kātyāyana*. Let us first of all discuss first the three rules given by *Baudhāyana*. These are :

आयामतृतीयेन त्रीणि चतुरस्राण्यनूचीनानि कारयेदपरस्योत्तरस्यां श्रोण्यां गार्हपत्यः तस्यैव दक्षिणेऽंसेऽन्वाहार्यपचनः पूर्वस्योत्तरेऽंसे आहवनीय इति। बौ. शु. १·६७·

"With the third part of the length (of the distance between *Āhavanīya* and *Gārhapatya*), he is to make three squares touching each other. The place of the *Gārhapatya* is in the North-West corner of the Western square, the place of the *Anvāhāryapacana* or (*Dakṣiṇāgni* is in the South-East corner of the same square; the place of the *Āhavanīya* is in the North-East corner of the Eastern square". Baudh Śl : 1.67.

Similar rules, though worded differently, are given by both *Kātyāyana* and *Āpastamba*. These are :—

अपि वाऽन्तः त्रिभागोनया रज्ज्वा पूर्वार्द्धे समचतुरस्रं कृत्वा श्रोण्यामग्निः। का.शु.१.२९.

"Or else, after having made a square, by a cord, which is reduced by a third of the distance between the *Gārhapatya* and the *Āhavanīya* and which lies in the exterior half on the East side of *Garhapatya*. One should place the *Dakṣiṇāgni* in the centre of the South-West corner of this square".Kāt : Śl : 1.29.

दक्षिणत. पुरस्ताद्वितृतीयदेशे गार्हपत्यस्य नेदीयसि दक्षिणाग्नेर्विज्ञायते।

आप. शु पृ. ६५.

"On the East of *Gārhapatya* and at a distance of *Vitṛtīya*, and from the line joining the centres of the two sacred fires, on the South is the place for the *Dakṣiṇāgni*". Āpast. Śl. page 65.

Fig. 37 is drawn as per rules quoted by both *Baudhāyada* and *Āpastamba*.

Fig. 38 is drawn as per rule stated by *Kātyāyana*.

A = place for *Āhavanīya*, G = place for *Gārhapatya*
and D = place for *Dakṣiṇāgn*.

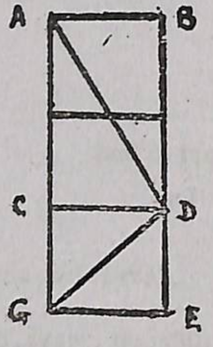

Fig 37

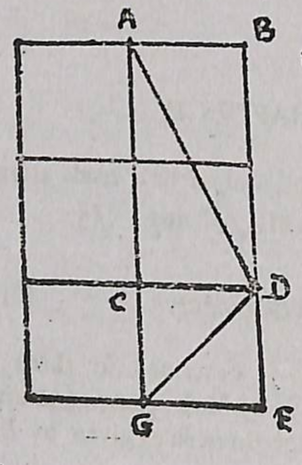

Fig 38

Fig 37 is formed by placing three equal squares side by side.

Fig. 38 is formed by placing a small square just touching a bigger one. Then the whole figure is divided into two parts-One a rectangle ABDC (formed by two squares) and a square CDEG.

∴ The diagonal AD of the rectangle ABDC will be equal to $\sqrt{5}$.

$$\therefore \quad AD^3 = AC^3 + CD^3 \qquad \because \quad AC = 2\,CD.$$
$$= (2\,CD)^3 + CD^3$$
$$= 4\,(CD^2 + CD^2$$
$$= 5\,CD^3$$

∴ The diagonal AD = $\sqrt{5} \cdot CD$.

Similarly the diagonal GD of the □ CDEG will be

$$GD^2 = CD^2 + CG^2 = 2\,CD^3.$$

$$\therefore \quad GD = \sqrt{2} \cdot CD.$$

Thus if the value of the length of the side of a square in the above case is known the values of $\sqrt{5}$ and $\sqrt{2}$ can be easily found.

Thus by making use of the sign of *vitṛtīya* (like the sign of *nirañchana*)on the total length of the cord and stretching the same on the South for fixing the place of *Dakṣiṇāgni*. it will be seen that an attempt for finding the values of the two surds $\sqrt{5}$ and $\sqrt{2}$ was made.

The distance between the centres of the *Āhavanīya* and *Gārhapatya* is given as 8, 11 or 12 *prakramas* by various authors as under :

आहवनीयोऽथ........गार्हपत्यस्तस्मादेतं.......गार्हपत्यात्प्राञ्चमुद्धरन्ति। २२

तं वा अष्टासु विक्रमेषु आदधीत ।२३। एकादशस्वादधीत .......।२४।

द्वादशस्वादधीत ।२५। शत.ब्रा. (१.७.३.२२-२५)

(अ) पुरस्तादाहवनीयस्याष्टासु प्रक्रमेष्वेकादशसु द्वादशसु मत्या वा।

का. श्रौ. सू. ४.८.१९.

(ब) अष्टासु ब्राह्मणस्यैकादशसु राजन्यस्य द्वादशसु वैश्यस्य।

सत्याषाढ. श्रौ. सू. ३.५.३.

(क) तस्मात् प्राचीनमष्टासु प्रक्रमेषु ब्राह्मणस्याहवनीयायतनम्। एकादशसु राजन्यस्य।
द्वादशसु वैश्यस्य। आप. श्रौ. सू. ४.२.२२.

Baudha.Śl. 1.66.

Similar statement is made by *Baudhāyana* also[1]. Now to locate the position of the *Dakṣiṇāgni* use of the *Vitṛtīya* mark is made. Let us see how the same is used.

---

(1) अष्टासु प्रक्रमेषु ब्राह्मणोऽग्निमादधीतैकादशसु राजन्यो द्वादशसु वैश्यः।
बौ. शु. १·६६.

**1st Method :**

अपि. वा गार्हपत्याहवनीययोरन्तरालं पञ्चधा षोढा वा संभुज्य षष्ठं सप्तमं वा भागमागन्तुकमुपसमस्य समं त्रैधं विभज्य पूर्वस्मादन्त्याद् द्वयोर्भागयोर्लक्षणं करोति । गार्हपत्याहवनीययोरन्तौ नियम्य लक्षणेन दक्षिणाऽऽयम्य लक्षणे शङ्कु निहन्ति तद्दक्षिणाग्नेरायतनं भवति । बौ. शु. १.६८.

"Or else divide the space between *Āhavanīya* and *Gārhapatya* either into five or six parts, add 1/6th or 1/7th part (according to the division you have Preferred) and divide the whole into three parts; then measuring from the pole standing at the East end, make a sign at the 2nd part of the cord, fasten the two ends of the cord at the poles marking the centres of the *Gārhapatya* and *Āhavanīya*, stretch the cord towards South taking it at the mark and fix a pole on the spot where the mark touches the ground. This is the place of *Dakṣiṇāgni*".
*Baudh*. Śl. 1.68.

Exactly similar rule, slightly differently worded is given by *Āpastamba*[1] and *Kātyāyana*.[2]

In short in this case, the length of the measuring cord, which is equal to the distance between the centres of the *Gārhapatya* and *Āhavanīya* is either increased by 1/6th or 1/7th and the total length of the increased cord so arrived at is then divided into three equal parts. A mark is then made on this lengthened cord at the end of the second part. This mark is cailled *vitṛtīya*.

Now taking the distance between the centres of *Gārhapatya* and *Āhavanīya* as 8, 11 and 12 units and then adding 1/6th or 1/7th to the above, let us try to find out the values of $\sqrt{3}$ and $\sqrt{2}$.

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(1) गार्हपत्याहवनीययोरन्तरालं पञ्चधा षड्धा वा संविभज्य षष्ठं सप्तमं वा भागम् आगन्तुमुपसमस्य समं त्रैधं विभज्यापरस्मिंस्तृतीये लक्षणं कृत्वा गार्हपत्याहवनीययोरन्तौ नियम्य लक्षणेन दक्षिणाऽपायम्य निमित्तं करोति । तद्दक्षिणाग्नेरायतनम् । श्रुतिसामर्थ्यात् । आप. शु. ६६.

(2) गार्हपत्याहवनी योरन्तरालं षड्ढा सप्तधा वाऽगन्तुसमं त्रेधा विभज्यापरत्रितृतीयलक्षणेन दक्षिणाऽऽयम्य तस्मिन्नग्निः । का. शु. १.२७.

(A)

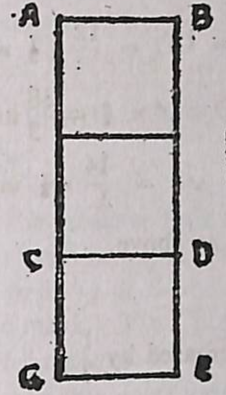

Fig 39

From Fig. 39 it will be seen that

$$AD = \sqrt{3} \cdot CD \text{ or } \sqrt{5} = \frac{AD}{CD}$$

$$GD = \sqrt{2} \cdot CD \text{ or } \sqrt{2} = \frac{GD}{CD}$$

(a) If AG = 8 then as per above rule

$$AG = 8 \times AC = 8 \cdot \frac{2}{3} = \frac{16}{3} \text{ and}$$

$$CG = CD = \frac{8}{3}$$

$$AD + DG = 8 + \frac{8}{6} = \frac{28}{3}$$

$$\therefore AD = \frac{28}{3} \times \frac{2}{3} = \frac{56}{9} \text{ and } GD = \frac{28}{3} \times \frac{1}{3} = \frac{28}{9}$$

$$\because \sqrt{5} = \frac{AD}{CD} = \frac{56}{9} \times \frac{3}{8} = \frac{7}{3} = 2.333 \quad \ldots \quad \ldots (1)$$

$$\text{and } \therefore \sqrt{2} = \frac{GD}{CD} = \frac{28}{9} \times \frac{3}{8} = \frac{7}{6} = 1.1666 \quad \ldots \quad \ldots (2)$$

(b) If AG = 11 then as per above.

$$AG = 11. \quad AC = 11 \times \frac{2}{3} = \frac{22}{3} \text{ and } CG = CD = \frac{11}{3}$$

$$\therefore AD = \sqrt{5}.CD \text{ or } \sqrt{5} = \frac{AD}{CD}$$

$$\text{and } GD = \sqrt{2}.CD \text{ or } \sqrt{2} = \frac{GD}{CD}$$

$$AD + DG = 11 + \frac{11}{6} = \frac{77}{6}$$

$$\therefore AD = \frac{2}{3} \times \frac{77}{6} = \frac{77}{9} \text{ and } DG = \frac{1}{3} \times \frac{77}{6} = \frac{77}{18}$$

$$\therefore \sqrt{5} = \frac{77}{9} \times \frac{3}{11} = \frac{7}{3} = 2.333 \quad \ldots \quad \ldots (1)$$

$$\text{and } \sqrt{2} = \frac{77}{18} \times \frac{3}{11} = \frac{7}{6} = 1.1666 \quad \ldots \quad \ldots (2)$$

(c) If $AG = 12$

$AC = 12 \times = \frac{2}{3} 8$ and $CG = CD = \frac{12}{1} \times \frac{1}{3} = 4$

$AD + DG = 12 + \frac{12}{6} = 14. \therefore AD = 14 \times \frac{2}{3} = \frac{28}{3}$ and $DG = 14 \times \frac{1}{3} = \frac{14}{3}$

$\therefore \sqrt{5} = \frac{28}{3} \times \frac{1}{4} = \frac{7}{3}$ and $\sqrt{2} = \frac{14}{3} \times \frac{1}{4} = \frac{7}{6}$

$\therefore \sqrt{5} = 2.333$ and $\sqrt{2} = 1{\cdot}1666$ as above.

**2nd Method**

(B) When the measuring cord is increased by $\frac{1}{7}$

(a) If $AG = 8$ then $AC = \frac{16}{3}$ and $CD = \frac{8}{3}$

$AD + DG = 8 + \frac{8}{7} = \frac{64}{7} \therefore AD = \frac{64}{7} \cdot \frac{2}{3} = \frac{128}{21}$

and $DG = \frac{64}{7} \times \frac{1}{3} = \frac{64}{21}$

$\therefore \sqrt{5} = \frac{128}{21} \times \frac{3}{8} = \frac{16}{7} = 2.2857$ .. .. (1)

and $\sqrt{2} = \frac{64}{21} \times \frac{3}{8} = \frac{8}{7} = 1.1428.5$ .. ... (2)

(b) If $AG = 11$ then $AC = \frac{22}{3}$ and $CD = \frac{11}{3}$

$AD + DG = 11 + \frac{11}{6} = \frac{88}{7} \therefore AD = \frac{88}{7} \times \frac{2}{3} = \frac{176}{21}$

and $GD = \frac{88}{7} \times \frac{1}{3} = \frac{88}{21}$

$\therefore \sqrt{5} = \frac{176}{21} \times \frac{3}{11} = \frac{16}{7} = 2{\cdot}2857$ ... ... ... (1)

$\sqrt{2} = \frac{88}{21} \times = \frac{8}{7} = 1{\cdot}14285$ ... ... ... (2)

(c) If $AG = 12$ then $AC = 8$ and $CD = 4$

$AD + DG = 12 + \frac{12}{7} = \frac{96}{7} \therefore AD = \frac{96}{7} \times \frac{2}{3} = \frac{64}{7}$

and $CD = \frac{32}{7}$

$\therefore \sqrt{5} = \frac{64}{7} \times \frac{1}{4} = \frac{16}{7}$ and $\sqrt{2} = \frac{32}{7} \times \frac{1}{4} = \frac{8}{7}$

i.e. $\sqrt{5} = 2{\cdot}2857$ and $-\overline{2} = 1{\cdot}14235$ as above.

(C) **3rd Method—**

The next method suggested by *Baudhāyāna* is as under :

अपि वा प्रमाणं पञ्चमेन वर्धयेत्तत्सर्वं पञ्चधा संभुज्यापरस्मादन्त्यात् द्वयोर्भागयोर्लक्षणं करोति। पृष्ठ्यान्तयोः पाशौ प्रतिमुच्य लक्षणेन दक्षिणापायम्य लक्षणे शङ्कुं निहन्ति। तद्दक्षिणाग्नेरायतनं भवति। बौ. शु. १.६९.

" Or else increase the measure by its fifth and divide whole into five parts. Make a mark at the end of two parts, measuring from the Western end; fasten the ties at the end of the *pṛṣṭhyā* or East-West line and stretch the cord towards the South taking it by the mark. Fix a pole at the place touched by the mark. This is the place of the *Dakṣiṇāgni*. Baudha. Śl. 1.69.

Divide the distance in five parts, increase the measuring cord by the fifth part and then divide the whole length of the cord into two parts—one consisting of three and the other of two parts.

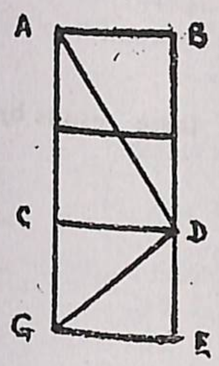

Fig 40

(a) If $AG = 8$ $AC = 8 \times \frac{2}{3} = \frac{16}{3}$

and $CG = CD = 8 \times \frac{1}{3} = \frac{8}{3}$

$AD + DG = 8 + \frac{8}{5} = \frac{48}{5}$

$\therefore AD = \frac{48}{5} \times \frac{3}{5} = \frac{144}{25}$

and $DG = \frac{48}{5} \times \frac{2}{5} = \frac{96}{25}$

Now $\sqrt{5} = \frac{AD}{CD} = \frac{144}{25} \times \frac{3}{8} = \frac{54}{25} = 2.16$ .. (1)

and $\sqrt{2} = \frac{GD}{CD} = \frac{96}{25} \times \frac{3}{8} = \frac{36}{25} = 1.44$ ... (2)

(b) If $AG = 11$ then $AC = 11 \times \frac{2}{3} = \frac{22}{3}$ and $CG = CD = 11 \times \frac{1}{3} = \frac{11}{3}$

$D + DG = 11 + \frac{11}{5} = \frac{66}{5} \therefore AD = \frac{66}{5} \times \frac{3}{5} = \frac{198}{25}$

and $DG = \frac{66}{5} \times \frac{2}{5} = \frac{132}{25}$

Now $\sqrt{5} = \frac{AD}{CD} = \frac{198}{25} \times \frac{3}{11} = \frac{54}{25} = 2.16$ .. (1)

and $\sqrt{2} = \frac{GD}{CD} = \frac{132}{25} \times \frac{3}{11} = \frac{36}{25} \doteq 1.44$ ... (2)

(c) Similarly if $AG = 12 \quad AC = 8$ and $CG = CD = 4$

$$AD + DG = 12 + \frac{12}{5} = \frac{72}{5} \quad \therefore \quad AD = \frac{72}{5} \times \frac{3}{5} = \frac{216}{25}$$

$$\text{and } DG = \frac{72}{5} \times \frac{2}{5} = \frac{144}{25}$$

$$\text{Now} \quad \sqrt{5} = \frac{AD}{CD} = \frac{216}{25} \times \frac{1}{4} = \frac{54}{25} = 2.16 \quad .. \qquad (1)$$

$$\text{and} \quad \sqrt{2} = \frac{GD}{CD} = \frac{144}{25} \times \frac{1}{4} = \frac{36}{25} = 1{\cdot}44 \quad \ldots \qquad (2)$$

Thus we get the following values as per above for $\sqrt{5}$ and $\sqrt{2}$.

$\sqrt{5} = 2{\cdot}3333; \quad 2{\cdot}2857; \quad 2{\cdot}16$

$\sqrt{2} = 1{\cdot}1666; \quad 1{\cdot}14285; \quad 1{\cdot}44$

According to present calculations the values of

$\sqrt{5} = 2.23607$ and $\sqrt{2} = 1{\cdot}414213.$

These values are approximate only.

The ancient Indians must have checked the accuracy of these results by squaring them. Thus

$\sqrt{5} = 2{\cdot}(3333)^2; \quad (2.2857)^2$ or $(2{\cdot}16)^2$

$= 5.44428889; \quad 5.3416809; \quad 4.6656$

and

$\sqrt{2} = (1.1666)^2; \quad (1.14285)^2$ or $(1.44)^2$

$= 1{\cdot}36095556: \quad 1{\cdot}3061061225; \quad 2.0736.$

It is most interesting to find, a similarity of procedure followed in both the cases of *nirañchana* and *vitṛtīya*. In these two cases :

(a) The original length of the measuring cord is increased by a fraction of the same.

(b) The increased length is then divided into two parts.

In the case of *nirañchana* it is (1) The hypotenuse and (2) The shorter side of the rt. ∠ed △.

But in the case of *vitṛtīya*, the increased cord is divided in lengths of $\sqrt{5}$ and $\sqrt{2}$; which are hypotenuses of two rt. ∠ed △s.

The procedure was based on the distinct understanding that in a square the hypotenuse produces an area which is twice the area of its side and in a rectangle the sides of which are in the ratio of 1 : 2, the hypotenuse of this rectangle will produce an area which is five times the area of the shorter side.

I have not come across the use of $\sqrt{5}$ in the construction of altars so far, The use of $\sqrt{2}$ is already explained.

In the case of *nirañchana* there were two rt. ∠ed △s,

(1) of units 3, 4 and 5 and (2) of units 5, 12, and 13 before them, Framing rules for these two only was very easy,

In the case of *vitṛtīya* such was not the case :

The procedure shows clearly that the ancient Indians had acquired a thorough knowledge of the Theorem of the Hypotenuse. The results obtained are no doubt approximate, which fact was known to them and hence an attempt to arrive at the result by some other method was already thought of and the same is explained in another article "To find the value of $\sqrt{2}$".

This is based on the Theorem of the hypotenuse of a square and the result obtained for the value of $\sqrt{2} = 1{\cdot}4142156$ was no doubt approximate but accurate upto 5 places of decimals.

(2) The commentator *Rāma* was an inhabitant of *Naimiṣa* (near modern Lucknow). He has written the commentaries on the Kātyāyana *Śulba* and *Śāradā-tilaka-Tantra*. His book *Kuṇḍākṛti* was composed in *Vikrama saṁvat* 1506 (1449 A.D.). The most notable contribution of his is a correction to the well known *Śulba* value of $\sqrt{2}$ namely :

$$\sqrt{2} = 1 + \tfrac{1}{3} + \frac{1}{3 \times 4} - \frac{1}{3 \times 4 \times 34} \qquad - \ (1)$$

*Rāma* shows that a more accurate (*Sūkṣma-tara*) value will be given by

$$\left[\sqrt{2} = 1 + \tfrac{1}{3} + \frac{1}{3 \times 4} - \frac{1}{3 \times 4 \times 34} - \frac{1}{3 \times 4 \times 34 \times 33} + \frac{1}{3 \times 4 \times 34 \times 34} \ldots (2)\right]$$

Turned into decimal fraction the expression (1) gives

$\sqrt{2} = 1{\cdot}4142156863$ .. (1) As per *Śulba*.

$\sqrt{2} = 1{\cdot}414213502$ .. (2) As per *Rāma*

$\sqrt{2} = 1{\cdot}41421356$ ... (3) As per modern calculations.

So that the value given by *Rāma* is correct upto seven places of decimals and that of *Śulba* upto 5 places of decimals.

(3) The value of $\sqrt{3}$ (*Trikaraṇi*) was obtained by making use of the Theorem of the hypotenuse. In this case one side of the rt. ∠ed △ was equal to $\sqrt{2}$ and the other side 1.

*Dwārakānātha Yajvā* states that

$\frac{10}{\sqrt{3}}$ = 5 *aṅgulas* and $27\frac{1}{2}$ *tilas*.

$\frac{12}{\sqrt{3}}$ = 6 *aṅgulas* and 32 *tilas*.

$\frac{15}{\sqrt{3}}$ = 8 *aṅgulas* and 23 *tilas*.

From these we get $\frac{1}{\sqrt{3}} = 0{\cdot}580, \quad 0.5784313723; \quad 0.57843137$

According to modern calculations = $\frac{1}{\sqrt{3}} = 0{\cdot}5773$

Thus it will be seen that attempts were being made to improve the values by later writers.

## CHAPTER V

### To Find the Value of $\sqrt{2}$

Where was the necessity of finding the value of $\sqrt{2}$.

This was found necessary for the following reasons—

(1) To construct a semicircle, equal in area to that of a circle.

(2) To construct an altar having an area; double (twice) or three times the first altar.

(3) To construct a square having an area of two sq. *puruṣas*.

**Definition of $\sqrt{2}$ :** All the *sūtrakāras* quote one and the same rule as a definition of $\sqrt{2}$. This is :

समचतुरस्रस्याक्ष्णया रज्जुर्द्विकरणी । का. शु. २.१२.

" In a square its diagonal will produce an area which will be double (twice that of its side ". Kāt. Śl. 2.12.

In other words, the length of the diagonal of a square = $\sqrt{2}x$, the side of the square. This is the rule stating the value of $\sqrt{2}$ : As $\sqrt{2}$ is an incommensurable quantity as explained in the previous article, the *sūtrakāras* defined its value by another rule which is as under :

करणीं तृतीयेन वर्धयेत्तच्च स्वचतुर्थेनात्मचतुस्त्रिंशोनेन सविशेष इति विशेषः ।

का. शु. २.१३.

" One should increase the measure (The side of a given square whose double producer is to be found) by its third part and again by the fourth part of this third, less by the 34th part of this fourth. The value thus obtained is called the *saviśeṣa*. This is the difference between a *Karaṇī* (The side of a square) and its *Dvikaraṇī* (The diagonal of the square) ". Kāt : Śl : 2.13.

$$\sqrt{2} = 1 + \tfrac{1}{3} + \frac{1}{3.4} - \frac{1}{3.4.34} = \frac{577}{408} = 1.4142156$$

The present mathematicians adopt the value of $\sqrt{2} = 1{\cdot}414213$.

The value of $\sqrt{2} = 1{\cdot}4142156$ as obtained by the ancient Hindus is correct upto 5 places of decimals.

The above rule is criticised by three eminent Western Scholars. Who are : (1) Dr. G. Thibaut, (2) Mr. T. L. Heath and (3) Mr. G. R. Kaye.

To their comments detailed answers are given by (1) B. B. Dutt and (2) Prof. L. V. Gurjar,

(A) I will first state the comments : These are :

Dr. G. Thibaut in his essay "On the *Śulbasūtras* " published in the journal Asiatic tfohe Society of Bengal : Vol. 44, part I—1875 writes as follows :

" The sūtras themselves are very short and do not state, which measure is to be increased ". The commentators on *śulba* state : That this measure is a *karaṇī*, the side of a square and the increased measure the diagonal (the *dvikaraṇī*).

" If the side of a square is 1, the diagonal of the square will be equal to $\sqrt{2} = 1{\cdot}414213$. Comparing this with the value of *saviśeṣa* (1·4142156) we can not fail to be struck by the accuracy of the latter ". Regarding the practical use of this *saviśeṣa* there is in all *sūtras* only one operation—of turning a circle into a square.

(B) Mr. T. L. Heath in his book entitled " The thirteen books of Euclid" published in 1908, writes with reference to the above essay by Dr. G. Thibaut and the translation of the *Āpastamba Śulbasūtra* written and published by Edmund Burk in Germany in the year 1901, as follows :

(1) It remains to consider Burk's claim that the Hindus had discovered the irrational : This is based on the value of $\sqrt{2}$ given by all the *śulbakāras*. There is nothing to show how this was arrived at. Dr. G. Thibaut's suggestion certainly seems best and the most natural. If in accordance with the old indian fashion a gnomon with unit area were to be subtracted from a square with 17 as a side; this would approximately be secured by giving the gnomon the breadth 1/34; i.e. 2 × 17 × 1/34 dividing out by 12 we have :

$$\sqrt{2} = 1 + \tfrac{1}{3} + \frac{1}{3 \times 4} - \frac{1}{3 \times 4 = 34} \text{ approximately.}$$

But it is a far cry from this calculation of an approximate value to the discovery of the irrational. First, we ask (1) Is there any sign that this value was known to be in exact ? (2) It comes directly after the statement that the square on the diagonal of a square is double that square. (3) The rule is quite boldly stated without any qualification.

Dr. Thibaut says the object of the whole proceeding was purely practical. He would without hesitation ignore this as being of no practical importance and the formula would be handed down and taken as a matter of course without arousing suspicion as to its accuracy. This supposition is confirmed by reference to rules quoted by *Āpastamba* for the construction of a circle equal in area to a square. Where the value of $\pi$ is taken as 3.09 and still he says it is exact "

Mr. T. L. Heath further continues : " Vogt says three stages had to be gone through before the irrationality of the diagonal of a square was discovered in any real sense (1) All the values found by direct measurement of calculations based thereon have to be as inaccurate. (2) Must supervene the conviction that it is impossible to arrive at an accurate arithmetical expression of the value and lastly (3) The impossibility must be proved.

(C) G. R. Kaye's Comments : G. R. Kaye has published a book in 1915 by name " Indian Mathematics " from Calcutta and Simla, through Thacker, Spink & Co. He says : " The result of the value of $\sqrt{2}$ is only an expression of a direct measurement and for a side of six feet, it is accurate to about 1/7th of an inch.

In answer to the above comments, first see what Dr. Thibaut says : (1) The *sūtras* are very short and do not state what measure is to be increased.

The meaning of the word *sūtra* is a concise or a short statement. As the rules were to be recited by heart, for want of paper and the art of printing the rules were prepared as concise as possible. In order to ascertain the proper meaning of the *sūtra* one has to see the sequence of the rules provided and hence the omission of the expression of the measure to be increased. It is a side of the square, as stated by the commentator and the value of the side of this square in this case is 1.

Mr. T. L. Heath's comments are : (1) Did the Hindus know that the value of $\surd 2$ was inexact ? (2) it directly follows the statement for the diagonal of a square. (3) it is a bold statement etc.

Before proceeding further let us see first as to what is the importance of the value of $\surd 2$.

In any square, whatever be the length of its side, the length of the diagonal is always $\surd 2$ times its side. (whether it is a complete number or a fraction).

For example .

| Side of the square | Square of the side | Value of the diagonal |
|---|---|---|
| 1 | 1 | $\sqrt{2} = \sqrt{2}$ |
| 2 | 4 | $\sqrt{8} = 2 \cdot \sqrt{2}$ |
| 3 | 9 | $\sqrt{18} = 3 \cdot \sqrt{2}$ |
| 4 | 16 | $\sqrt{32} = 4 \cdot \sqrt{2}$ |
| 5 | 25 | $\sqrt{50} = 5 \cdot \sqrt{2}$ |
| 7 | 49 | $\sqrt{98} = 7 \cdot \sqrt{2}$ |
| 12 | 144 | $\sqrt{288} = 12 \cdot \sqrt{2}$ |
| 17 | 289 | $\sqrt{578} = 17 \cdot \sqrt{2}$ |
| $\frac{1}{2}$ | $\frac{1}{4}$ | $\sqrt{\frac{1}{2}} = \frac{1}{2} \cdot \sqrt{2}$ |
| $\frac{1}{3}$ | $\frac{1}{9}$ | $\sqrt{\frac{2}{9}} = \frac{1}{3} \cdot \sqrt{2}$ |

(a) In the above examples $7^2$ (49) is less by one in the case when the side of the square is 5, and the square of the length of the diagonal is 50.

(b) Similar is the case when $17^2 = 289$ is by 1 more than the length of the square of the diagonal $2 \times 12^2 = 288$ when the side of the square is 12.

i.e. $17^2 - 2.12^2 = 1^2$. or $7^2 - 2\,(5)^2 = -\,1^2$.

**According to various scholars the above relation is made use of for finding the value of $\surd 2$, by *sūtrakāras*.**

Thus when the side of a square is 12, the value of the length of the diagonal will be $\surd 288$, this is less by 1 than $17^2 = 289$, i.e. the length of the diagonal will be less than 17.

This can be found by three different ways :—

(1) Deduct a square from another square. Kāt : Śl : 3.1.

(2) To find the sum of any number of squares. Kāt. Śl. 6.7.

(3) The third one is already quoted above.

But in the case of the above two methods one has practically to measure the lengths and find the solution, they must have found the solution as approximate as possible by giving certain values to the sides : Thus if 1 *aṅgula* = 16 *tilas*

(A) 1 *aṅgula* = 16 *tilas*. $\therefore$ 17 *aṅgulas* $= 17 \times 16 = 272$ *tilas*.

$\therefore$ $272 - 1 = 271$, $\therefore$ $271^2 = 73441$ sq. *tilas*.

Now $12 \times 16 = 192$ *tilas*. $\therefore$ $2 \times 192^2 = 2 \times 36864 = 73728$ sq, *tilas*

i.e. the result obtained is $(73441 - 73728 = 287)$ more by 287.

(B) 1 *aṅgula* = 20 *tilas*.

$\therefore$ $17 \times 20 = 340$ and $340 - 1 = 339$. $\therefore$ $339^2 = 114921$ sq. *tilas*.

and $12 \times 20 = 240$ $2 \times 240^2 = 2 \times 57600 = 115200$ sq. *tilas*.

$\therefore$ The result is less by $115200 - 114921 = 279$.

(C) 1 *aṅgula* = 24 *tilas*.

$\therefore$ $17 \times 24 \equiv 408$ $\therefore$ $408 - 1 = 407$ $\therefore$ $407^2 = 165649$ sq. *tilas*.

now $12 \times 24 = 288$ $\therefore$ $2 \times 288^2 = 2 \times 82943 = 165886$.

$\therefore$ The result is less by $165886 - 165649 = -\,237$.

(D) 1 *aṅgula* = 30 *tilas*.

$\therefore$ $17 \times 30 = 510$ $\therefore$ $510 - 1 = 509$ $\therefore$ $509^2 = 259061$.

and $12 = 30 = 360$ $\therefore$ $2 \times 360^2 = 2 \times 129600 - 259200$,

$\therefore$ The result is less by $259200 - 259061 = -\,119$.

(E) 1 *aṅgula* = 34 *tilas*.

$\therefore$ $17 \times 34 = 578$ $\therefore$ $578 - 1 = 577$. $\therefore$ $577^2 = 332929$.

and $12 \times 34 = 408$ $\therefore$ $2 \times 408^2 = 2 \times 166464 = 332928$.

$\therefore$ The result is one more than desired $(332929 - 332928 = 1)$.

(F) 1 *aṅgula* = 36 *tilas*.

$\therefore$ $17 \times 36 = 612$ $\therefore$ $612 - 1 = 611$ $\therefore$ $611^2 = 373321$.

and $12 \times 36 = 432$ $\therefore$ $2 \times 432^2 = 2 \times 186624 = 373248$.

The result is more $(373321 - 373248 = 73)$ by 73.

**Thus the result obtained by taking 1 *aṅgula* = 34 *tilas* is a nearer approximation desired.**

Regarding (1) please see the answers given by both Shri B. B. Dutta and **Prof. L. V. Gurjar.**

Regarding items (2) and (3) please read the answer to the general comment of Dr. Thibaut that both the rules regarding the diagonal of a square and a rectangle must be considered together so as to get at the proposition of the hypotenuse as described by Pythagoras.

B. B. Dutta's answer : B. B. Dutt in his book " The science of *Śulba* or the Ancient Indian Geometry " published in the year 1932, by the University of Calcutta writes as follows :

Did the ancient Hindus recognise the irrationality of $\sqrt{2}$? This question does not seem to have troubled Thibaut in any way. For we do not find him to raise it explicitly or to attempt to answer it directly. But it is clear from his writings that he believed in the Hindu knowledge of the irrationality of $\sqrt{2}$. Indeed his theory about the discovery of the Hindu approximation to the value of $\sqrt{2}$, which has been quoted before, is fundamentally based on the knowledge of the incommensurability of the diagonal of a square with its sides. Thibaut supposes that the ancient Hindus endeavoured to discover a square the side and diagonal of which might be expressed in integrals and then observes :

" Undoubtedly they arrived soon at the conclusion that they would never find exactly what they wanted and had to be contented with an approximation ".

Von Schoroeder and Burk are more explicitly emphatic. They claim for the ancient Hindus the credit for the first discovery of irrationals. And they have been followed in this respect by Garbe, Hopkins and Macdonell. But this hypothesis has been criticised and opposed by some modern historians of Mathematics.

The value arrived at, which is according to Dr. Thibaut is " less by 1/1156 " or in the opinion of Kaye " for a side of six feet, it is accurate to about 1/7th of an inch ". These statements of these two writers clearly show that the value derived by the rule of *Śulba* is not exact but only approximate.

With this much introduction B. B. Dutt shows how Dr. Thibaut has wrongly interpreted the word ' *saviśeṣa* '. He says : There are two terms which have undoubtedly a great bearing on the point under discussion. They are *viśeṣa* and *saviśeṣa*. Importance of these terms has not been properly realised by previous writers. (1) Thibaut simply observes that *saviśeṣa* is a technical name of the increased measure. (2) Burk remarks " The total increase is *viśeṣa* because it is the difference between the *pramāṇa* i,e, the side of the given square and its *dvikaraṇī*. Therefore, this latter is *saviśeṣa* with the difference.

In the *Śulba*, the calculated value of the diagonal of a square is technically called the *saviśeṣa* of its side or symbolically :

$$\textit{Saviśeṣa} \text{ of } a = a + \frac{a}{3} + \frac{a}{3 \times 4} - \frac{a}{3 \times 4 \times 34}$$

Now what is the radical significance of the term *saviśeṣa*? Before answering this question, we shall point out that occasionally the term has been used to

denote the complete diagonal in general; that is in the sense *saviśeṣa* of $a = a\sqrt{2}$. Again in one instance in the *Āpastamba Śulba*, we find the use '*Viśeṣa*' of :

$$a = \frac{a}{3} + \frac{a}{3 \times 4} - \frac{a}{3 \times 4 \times 3}$$

But on several occasions in this work and also in other *Śulbas*, particularly in a compound word the term *viśeṣa* has been employed in the sense of the hypotenuse of a right angled triangle. And it has again been considered there as equivalent to *saviśeṣa*. The concluding portion of the *Kātyāyana's* statement of the rule for *saviśeṣa* runs thus :—

'*Saviśeṣa iti viśeṣaḥ*'. Here the word *viśeṣa* must be explained differently. The use of the same terms thus in varied significations has made the interpretation of their origin really difficult. Thibaut's explanation is obviously erroneous. Burk's is not full.

Regarding the translation of the general rule there is no question of difference except the meaning of the word *saviśeṣa*.

(A) Dr. Thibaut translates the same as under :

" The name of the increased measure is *saviśeṣa*."

(B) Burk remarks : " The total increase is *viśeṣa* because it is the difference between the *pramāṇa* i.e. the side of the given square and its *dvikaraṇī* Therefore, this latter is *saviśesa* with the difference.

(C) *Kapardiswāmī* observes that the *saviśeṣa* is the technical term for the sum obtained as per general rule. As it is accompanied with a special quantity in excess (*viśeṣa*)[1].

It is a term whose meaning is intelligible. The square of *tilas* in 12 *aṅgulas* is 1,66,464 sq. *tilas*. Twice this is 2 × 166464 = 332928 sq. *tilas*. The square of *tilas* in 17 *aṅgulas* minus 1 *tila* is equal to 17 × 34 = 578 *tilas*. 578 − 1 = 577 *tilas*. ∴ $577^2$ = 002929 sq. *tilas*. In this latter a square of one *tila* is in excess. *Saviśeṣa* is the technical name for the whole quantity obtained as per rule, which is in excess by one sq. *tila*. So the formula is without fault. It will be of practical use. Taking this into consideration the learned author has made use of this term.

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[1] सविशेषः–सविशेष इति संज्ञा। एवं सवर्गितस्य सह विशेषेण वर्तत इति अन्वर्था संज्ञा। द्वादशे चत्वारि चतुर्षु एकं चतुस्त्रिंशद्वा विभज्य एकं भागमुत्सृज्य, नवं तिलोनसप्तदशाङ्गुलं भवति। द्वादशाङ्गुलस्य तिलवर्गः, एकं नियुतं षड्युतानि षट्सहस्राणि चत्वारि शतानि षष्टिः चत्वारीति (१,६६,४६४) तिलप्रमाणानि। सप्तदशाङ्गुलमानस्य तिलोनस्य तिलवर्गः–त्रीणि नियुतानि त्रीण्ययुतानि द्वे सहस्रे नव शतानि विंशतिर्नव च (३,२२,९२९), अत्रैक त्रिकयोश्शेषो न भवतीति वेणोस्सविशेषे गृह्यप्रमाणे दशतिलक्षेत्राण्यतिरिक्तानि। तेन नीवारशूकार्धमात्रमप्यतिरिक्तं भवति। तस्मात् विशेष इति व्यवहारार्थमेव भविष्यति इति प्रगणय्य संज्ञा कृताऽऽचार्येण। (कपर्दिभाष्यम्) आ. शु. प. २७.

(D) The following examples give the use of the word *viśeṣa* in the *Śulba sūtra* elsewhere. It's meaning differs.

(1) तासां द्वे अर्धेष्टके बाह्यविशेषे चुबुक उपदध्यात्। बौ. शु. ३.१६४.

(1) "From among these bricks, two half bricks their hypotenuse turned towards the outside, are to be placed in the *Chin*, BaudhŚl. 3.164.

Here *viśeṣa* = hypotenuse.

पृष्ठ्यान्तयोर्मध्ये च शङ्कुं निहत्यार्धेऽर्धे तद्विशेषमभ्यस्य लक्षणं कृत्वार्धमागमयेत्। अन्त्ययोः पाशौ कृत्वा मध्यमे सविशेषं प्रतिमुच्य पूर्वस्मिन्नितरं लक्षणेन दक्षिणमंसमायच्छेत्। आप. शु. पृ. ३३-३४.

"Add to half the length of the East-West line its *viśeṣā* ". Āpa. Śl.page 34

अवशिष्टं षोडशीभिः प्राच्छादयेत। अन्त्या बाह्यविशेषा अन्यत्र शिरसः।

आप. शु. पृ. २७५

Cover the remaining portion of the fire altar with *ṣodaśīs* (bricks). Those lying at the extremity will have their *viśeṣa* outwards except i.e. inwards at the heads".

*Viśeṣa* = inclined face of the brick = Hypotenuse. Āpa. Śl. page 275.

अपरस्मिन् प्रस्तारे पुरस्ताच्छिरसि द्वे षोडश्यौ बाह्यविशेषे उपदध्यात्। आ.शु.पृ.२७५

"In the 2nd layer, at the head towards the East lay two *ṣoḍaśīs* with their *viśeṣa* outwards and on the West lay them with their *viśeṣa* inwards and lying in both places (viṣaya, partly in) the head and partly in the body of the fire altar". Āpa. Śl. page 275.

षोडशी चतुर्भिः परिगृह्णीयात्। अष्टमेन त्रिभिरष्टमैश्चतुर्थेन चतुर्थं सविशेषेणेति।

आ.शु.पृ. २६४-२६५.

"With ½ *puruṣa* and *saviśeṣa* of ½ *puruṣa*". Āpa. Śl. page 265.

अर्धपदेन पदेनाध्यर्धपदेन पदसविशेषेणेति। बौ. शु. ३.६७.

"With half a *pada*,a *pada*, a *pada*; and a half and the *saviśeṣa* of a *pada*" Baudh. Śl. 3.67.

The *saviśeṣa* of a *pada* $= 15+5+\frac{5}{4}-\frac{5}{4.34}$ *aṅgulas*

Here *saviśeṣa* $= a+\frac{a}{3}+\frac{a}{3\times 4}-\frac{a}{3\times 4\times 34}$

अर्धप्रादेशेनाध्यर्धप्रादेशेन प्रादेशसविशेषेणेति। बौ. शु. ३.१४९.

"With half the *prādeśa*, a *prādeśa* and a half; a *prādeśa* and the *saviśeṣa* of *prādeśa*". Baudh. Sl. 3.149.

अध्यर्धेष्टकां चतुर्भिः परिगृह्णीयादर्धव्यायामेन द्वाभ्यामरत्निभ्यामरत्निसविशेषेणेति।

बौ. शु. ३.१५०.

"An *adhyardhā* brick is enclosed with four sides : Half a *vyāma*, two sides of one *aratni* length and the *saviśeṣa* of one *aratni*". Baudh. Śl. 3.150.

From the above examples it will be clear that the term *saviśeṣa* is used in the following different ways :

(a) *Saviśeṣa* = Calculated value of the diagonal of a square.

(b) *Saviśeṣa* = $a\sqrt{2}$; to denote the complete diagonal.

(c) In *Āpastamba* in one place,

$$Viśeṣa \text{ of } a = \frac{a}{3}+\frac{a}{3\times 4}-\frac{a}{3\times 4\times 34}$$

(d) *Saviśeṣa* = Viśeṣa.

Let us see now what *Karawindaswāmī* has to say on this point :

(1) (करविन्दीय व्याख्या) अत्र सह विशेषेण वर्तत इति सविशेषः। विशेषसहितं प्रमाणमेव सविशेष इत्यवगन्तव्यम्। किमर्थं महती संज्ञा अन्वर्थैवा यथा स्यादिति। तत्कथं विपूर्वशिषिरतिशयवाची। "शिषिरसर्वोपादाने विपूर्वोऽतिशये" इत्युपदेशात्। अक्ष्णयारज्जुभूतद्विकरण्याः प्रमाणातिरिक्तादंशात् किंचिद्विशिष्यते अतिरिच्यत इति विशेषः।

तथा हि–चतुस्त्रिंशत्तिला अङ्गुलिरित्युक्तम्। द्वादशाङ्गुलस्य तिलवर्गः—एकं नियुतं षड्युतानि षट् सहस्राणि चत्वारि शतानि षष्टिश्चत्वारि च दिलप्रमाणानि चतुरस्रक्षेत्रप्रमाणानि। तिलोनसप्तदशाङ्गुलस्य तिलवर्गः—त्रीणि नियुतानि त्रीण्ययुतानि द्वे सहस्रे नव शतानि विंशतिर्नव च तिलप्रमाणानि चतुरस्रक्षेत्राणि। अत्रैकं तिलं चतुरस्रक्षेत्रद्विगुणादतिरिच्यते ततस्तस्य विशेष इति संज्ञा। आप. शु. प. ३१.

"The root *śiṣ* when prefixed by *vi* denotes in all cases "a correction in excess". As the rule mentions of an excess qualified by the prefix *vi* it should be understood that the accurate value of the diagonal differs by something from, exceeds over the exceeding part of the above value of *dvikaraṇī* over the measure of the side. And that is the Viśeṣa; or else the Viśeṣa is that little area by which the square of the diagonal as calculated by rule, differs from or exceeds over twice the area of the given square at the time of measurement. According to the rule quoted by all the *Śulbas* a square of one *tila* is in excess over twice the area of the given square. Hence its technical name is Viśeṣa".

If the measure falls short, the area described in that case will also be smaller, so for doubling a square the *saviśeṣa* is employed as practical means. Commentator *Sundararāja* also observes in the same way.[1]

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(1) योऽक्षणयारज्ज्वा प्रमाणादतिरिक्तोंऽशः स विशेषसंज्ञः। आप. शु. पृ. ३३. यद्वा ततः किंचित् क्षेत्रं विशेषयति अतिरेचयति मानकाले इति वा विशेषः।

Now let us see whether the above view is confirmed by any other writings of the same period.

(1) In the *Sūryaprajñapati* (C. 500 B.C.). The circumference of a circle whose diameter is 99640 *yojanas* is stated as 315089 *yojanas* and a little over (*Kiñcid viśeṣādhika*).

(2) That of a circle of 100660 *yojanas* is stated to be 318315 and a little less. (*Kiñcid viśeṣeṇa*),

(3) In the *Jambūdvīpaprajñapati* (C. 300 B.C.) the circumference of the *jambūdvīpa* which is of the shape of a circle of 1;00,000 *yojanas* in diameter is mentioned as 3,16,237 *yojanas*, 3 *gavyūtis*, 128 *dhanus*, 13½ *aṅgulas* and a little over (*Kiñcid viśeṣādhikā*).

(4) In the *Trilokasāra* of *Nemicandra* (975 A. D.) the word *saviśeṣa* is exactly used in the same sense. He writes *Tallīnam parirayena saviśeṣam* or thrice that with a little over (*saviśeṣa*) is the circumference.

(a) in all these cases it will be seen that the value actually recorded is only an approximate one and the *Viśeṣa* refers to a small quantity which has not been recorded–in fact it cannot be accurately determined–but which we shall have to add or subtract from the recorded value in order to get the accurate value of the quantity sought.

(b) Those who are acquainted with the technique of Sanskrit and other sanskritic languages, will at once recognise that the expression *Kiñcid viśeṣādhikā* has the identical significance as that with which it is found in the early works of Jains. Thus it is found that the interpretation of the commentators *Karavindaswāmī* and *Kapardiswāmī* of the word *saviśeṣa* is correct.

From the above statements, one can come to the following conclusions :

(1) The meaning of the word *saviśeṣa* (that is the name of the increased measure) as interpreted by Dr. Thibaut is entirely wrong.

(2) If the *sūtrakāras* really wanted to use the same value (which is apparently approximate) they would not have used the word *saviśeṣa* along with the general rule which gives the value of $\sqrt{2}$ or the irrational.

(3) The words *Viśeṣa* as well as *Saviśeṣa* are used elsewhere in the *sūtra* itself, there the words differ in the meaning.

(4) The meaning of the word *Saviśeṣa* as used in the Jain literature in the Sanskrit language means "slightly in excess".

(5) All the commentators agree that the word *saviśeṣa* means the same thing i. e. slightly in excess.

(6) The word *saviśeṣa* was therefore added to the general rule to indicate that the value arrived at by the general rule was "slightly in excess" but not accurate and hence approximate.

Thus it is evident that according to the interpretation of the commentators *Kapardiswāmī* and *Karvindaswāmī* the term *saviśeṣa* for the diagonal of a square implies intrinsically the knowledge of the following :

(1) The value of the diagonal as calculated by the rule stated for the purpose is only an approximate one.

(2) That the value of the diagonal has a small quantity in excess over the true value of the diagonal or in the other words the square of that value is more by some quantity than twice the area of the given square.

(3) That excess cannot be completely eliminated in calculating the value of the diagonal arithmetically.

If the validity of the above interpretation is accepted there will remain nothing to doubt that the ancient Hindus were aware of the incommensurability of the diagonal of a square with its sides.

So far an attempt was made to show that the value of $\sqrt{2}$, obtained as per rule given in the *śulba* is not accurate but only approximate.

Prof. Gurjar deals on this subject in a different manner, which is most interesting. He says : The value of $\sqrt{2}$. when worked out would be found to be correct to five places of decimals. The question is how is it arrived at? The explanation given by Prof. Thibaut as to the probable method of calculations the *sūtrakāras* might have adopted fails to satisfy us.

According to him these calculations are based where the number expressing the square of the diagonal of a square differs only by one from a square number. The square of the diagonal of a square whose side is 12 is 288. The square of number 17 is 289. So the diagonal of a square whose side 12 will be nearer to 17.

If the sides of the squares could not be shortened by so much so that its area would contain not 289 but 288 such small squares, then the measure of the side would be the exact measure of the diagonal of a square, the side of which is equal to $12 \cdot (12^2 + 12^2 = 288)$, When the side of the square is shortened a little, the consequence is that from the sides of the square, a strip is cut off, therefore, a piece of that length had to be cut off, so that the area of the two strips would be equal to one of the 289 small squares.

Now as the square is composed of 17 × 17 squares, one of the two stripes cuts off a part of 17 small squares and the likewise of 17, both together 34 and since these cut off 34 pieces are to be equal to one of the small squares, the length of the piece to be cut off from the side is fixed. It must be 34th part of the side of one of the 289 small squares.

The 34th part of the 34 small squares being cut off and the area of the large square reduced to 288 small squares, the two stripes which are cut off from the sides of the square. Let us say the East-side and the South-side. intersect or overlap each other into the smallest corner and the consequence is that from the small square in that corner, not $\frac{2}{34}$ are cut off but only $\frac{2}{34} - 34 \times 34$, hence the error in the determination of the value of *saviśeṣa*.

In justification of his ingenious interpretation Prof. G. Thaibaut quotes an example given by the commentators of the *sūtras* viz., the square whose side is 12 and the diagonal is $16\frac{33}{34}$.

$$\text{For diagonal} = (12 + \frac{12}{3} + \frac{12}{3 \times 4} - \frac{12}{3 \times 4 \times 34}) \text{ approximately.}$$

But this example given by the commentators of later date is indeed the case recommended itself by being first in which the third part of the number and the fourth part of the third part are both whole integral numbers.

However, this clever and ingenious explanation by Thibaut fails to give the reason of the fraction $\frac{1}{3}$ and $\frac{1}{12}$ which occur as the second and third term. It can no doubt make a suggestion (though not plausible) for the presence of the number 34.

G. R. Kaye states (Indian Mathematics by Kaye) that "given scale of measure based upon the change ratio 3, 4 and 34 (since they had a scale of 34 *tilas* = 1 *aṅgula*), the result is only an expression of direct measurement. In my opinion Mr. Kaye has put the cart before the horse. Even accepting such an erroneous statement, the question still remains why did they choose such a scale or what led them to have such a scale of units?

Prof. Thibaut observes "Nothing in the *sūtras* would justify the assumption that they were expert in long calculations" and Kaye conveniently quotes only these words of Thibaut to qualify his statement and purposely avoids a further statement by Thibaut which is not only an answer to Kaye, but Thibaut thereby refutes his own statement. For he further says, : as the diagonal of a square is in reality an incommensurable quantity, we can expect an approximate value but their approximation is a **remarkably closed one.** (The bold type is author's statement). No doubt in *sūtras* they did not try to exhibit their expertness in long calculations, but it must be noted that *Śulbasūtras* are not mathematical works but works that had their origin in religious duties performed and as such there was no need of parading the above mentioned quality. They gave the value of $\sqrt{2}$ correct to five places of decimals, and **in those days it was really an un-** **common achievement.** How did the scale 34 *tilas* = 1 *aṅgula* and 12 *aṅgulas* = 1 *prādeśa* come into existence? Out of all the numbers, why the number 34 was suggested to *Baudhāyana*–The author of the oldest *Śulbasūtra* to define the measure *aṅgula* instead of any round figure? It is in fact, *Baudhāyana* came in the calculations of $\sqrt{2}$, across the 34th part of 12 that led him to introduce his scale of relation. *Baudhāyana* **adopted the system of units befitting the scienti-** **mind that harnesses theory with practice.** For instance, why is the metric

system of units introduced? Because it facilitates calculations in scientific experiments. The same genius that introduced the metric system was present in *Baudhāyaaa* when he used the value of $\sqrt{2}$ in defining his units and their measures. This is, therefore, the first attempt on the part of *Baudhāyana* to make the theory suit the practice and full credit must go to him for its priority.

It appears to my mind that his value of $\sqrt{2}$ must have been arrived at by the method of approximations, where the quantities of higher order of smallness are neglected or by the method which wrongly goes under the name "Tannery's R Process",

In this connection the reader must first of all note down an earlier *sūtra*.

Translation : A cord of the length of $1\frac{1}{2}$ *puruṣa* makes $2\frac{1}{4}$ sq. *Puruṣas*.

This shows that the *sūtrakāras* knew that the area of the square whose side is $(1 + \frac{1}{2})$ units is $2\frac{1}{4}$ sq. units.

i. e. $(1 + \frac{1}{2})^2 = 2\frac{1}{4} > 2$.

Hence they tried the square of $(1 + \frac{1}{3})$

But $(1 + \frac{1}{3})^2 = 16/9 < 2$.

$2 - 16/9 = 2/9$.

So they further took the quantity x so that

$$\left(1 + \frac{1}{3} + X\right)^2 = \frac{16}{9} + \frac{8X}{3} + X^2$$

$X^2$ They omitted as the quantity of higher order of smallness, hence puting

$$\frac{8X}{3} = \frac{2}{9} \qquad \text{We have } X = \frac{1}{12} = \frac{1}{3 \times 4}$$

$$\therefore \quad \sqrt{2} = 1 + \tfrac{1}{3} + \frac{1}{3 \times 4} \text{ as the 2nd approximation.}$$

$$\text{Further} \quad (1 + \tfrac{1}{3} + \frac{1}{3 \times 4} + Y)^2 = (\frac{17}{12} + Y)^2 = \frac{289}{144} + \frac{34Y}{12} + Y^2$$

$Y^2$ they omitted and hence putting.

$$\frac{34Y}{12} = -\frac{1}{144} \quad \text{we have}$$

$$Y = -\frac{1}{3 \times 4 \times 34}$$

$$\therefore \quad \sqrt{2} = 1 + \tfrac{1}{3} + \frac{1}{3 \times 4} - \frac{1}{3 \times 4 \times 34}$$

Or they might have followed another method that consists of spliting the quantity under the radical sign into two parts—One complete square and the other the remaining part i.e. the method that gives :—

$$\sqrt{A^2+b} = A + \frac{b}{2A} \text{ approximately.}$$

Thus $\sqrt{2} = \sqrt{1+1} = 1 + \frac{1}{2}$ as the first approximation.

But they knew that

$$\left(1+\frac{1}{2}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4} > 2$$

$\therefore$ They tried $\left(1+\frac{1}{3}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$

But $\frac{16}{9} < 2$ by $\frac{2}{9}$

$$\therefore \sqrt{2} = \sqrt{\frac{16}{9}+\frac{2}{9}} = \sqrt{\left(\frac{4}{3}\right)^2+\frac{2}{9}}$$

$$= \frac{4}{3} + \frac{\left(\frac{2}{9}\right)}{2\times\frac{4}{3}} = \frac{4}{3} + \frac{1}{3\times 4}$$

$$= 1 + \frac{1}{3} + \frac{1}{3\times 4}\ldots\ldots \text{ as the 2nd approximation.}$$

$$= \frac{17}{12}.$$

And $\left(\frac{17}{12}\right)^2 = \frac{289}{144}$. But $\frac{289}{144} < 2$ by $\frac{1}{144}$

$$\therefore \sqrt{2} = \sqrt{\frac{289}{144}-\frac{1}{144}} = \sqrt{\frac{(17)^2}{12}-\frac{1}{144}}$$

$$= \frac{17}{12} - \frac{\frac{1}{144}}{2\times\frac{17}{12}} = \frac{17}{12} - \frac{1}{3\times 4\times 34}$$

$$= 1 + \frac{1}{3} + \frac{1}{3\times 4} - \frac{1}{3\times 4\times 34} \text{ as the third approximation and so on.}$$

Had they required further approximations they would have proceeded in the same manner but they did not find it necessary, as this value of *saviśeṣa*, i.e. $\sqrt{2}$, served their purpose quite all right.

The second method is but a little variation of the first but it must have been this very method by which the *Sūtrakāras* obtained the value of $\sqrt{2}$ as it is in conformity with the word *saviśeṣa*.

*Saviśeṣa* = *Sa* + *vi* + *śeṣa*.

*Śeṣa* Means remainder or anything left out to be said.

*Vi* = As a prefix to nouns not immediately connected with roots expresses negative sense and hence *viśeṣa* means without anything being left out to be said i.e. complete in itself–i.e. a complete sq.

*Sa* = The prefix attached to nouns means " the same " " The celebrated "; " the distinguished "

Hence *Saviśeṣa* means the distinguished or the celebrated complete result. विशेषित : शेषः तेन युक्तं—सविशेषितशेषः or सविशेषविशेषः or for brevity सविशेषः.

Thus in $\sqrt{2}$ – $\frac{16}{3}$; $\frac{289}{144}$ are the *viśeṣa* or perfect squares, with 1; $\frac{2}{9}$ – $\frac{1}{144}$ as the corresponding *śeṣa* or remainders.

For example in $\sqrt{41}$; i.e. $\sqrt{36+5}$
36 is *viśeṣa* and 5 is *śeṣa*.

Thus it is evident that the process

$$\left[\sqrt{A^2+b} \sim \left(A+\frac{b}{2A}\right)\right]$$

must be known because of its priority as the *saviśeṣa* process and not as Tannery's R process. *Saviśeṣa* process is at least as old as 750 B. C., and bears a strong testimony that the Hindus had made enormous strides in the science of Mathematics in ancient days.

There is no wonder that the rule which was first given before 750 B. C. should be so extensively used and be further developed with the application of continued fractions in *Bakśāli* Manuscript.

We can legitimately place the work in a period extending from 200 B. C. to 200 A. D.

If the *Sūtrakāras* have applied the rule of approximations to find the value of $\sqrt{2}$, they called *saviśeṣa*, at the time of the *Bakśāli* composition. The rule was made more elaborate and was utilised to obtain the square root of any number to any degree of accuracy by taking approximations in succession. ( Extracts from the " Ancient Indian Mathematics and the Vedha ". by Prof. L. V. Gurjar ).

B. B. Dutt while describing the development of the value of $\sqrt{2}$ has laid more stress on the meaning of the words *sovi śeṣa* and *viśeṣa*. In order to show that what he says is correct (a) he first quotes the commentators (*Kapardiswamī*, *Karavindaswamī* and *Sundararāja* ) of the *Āpastamba śulba* for the meaning of the words quoted above. (b) He then shows the use of the same words in the *śulba* proper with different meanings. (c) Similar words are used in the Jain literature in sanskrit, of the same period, having the same meaning as the commentators and lastly he concludes that those words have a difinite meaning—The quantity calculated as per rule given in the *śulba* is not accurate; i e. it is approximate and is very slightly in excess.

Prof : Gurjar on the other hand lays stress on the Mathematical development of the value of $\sqrt{2}$.

(1) First of all he shows that there is a small error in the calculations of Dr. Thibaut.

(2) No relation between the numbers 12 and 34 has been properly established. Then he proceeds to show :

(3) That the value of $\sqrt{2}$ must have been derived at by the method of approximations.

Then with a slight variation in the above process he shows :

(4) That the method adopted was similar to the one now commonly known as Tannery's process.

In support of his statement he shows that :

(5) The same method was later on used for finding the square root of any number to any degree of accuracy at the time of *Bakšāli* Composition.

Now looking to the comments of Dr. T. L. Heath, it will be seen that the most important point to be proved was whether the ancient Hindus knew that the value of $\sqrt{2}$ was not accurate but only approximate.

In reply the first thing of note is that :

(1) The ancient Hindus had no knowledge of the present arithmatical process of finding the value of $\sqrt{2}$.

(2) The meaning of the word "*saviśeṣa*" clearly indicates that the value obtained as per rule given in the *Śulba* was slightly in excess i. e. it was not accurate but only approximate.

(3) The mathematical process, which gives a clue for finding the rule for fixing the value of $\sqrt{2}$ is based on approximations. Therefore the value so obtained from the rule can not be considered as accurate.

(4) The additional extra word "*saviśeṣa*" would not have been attached at the end of the rule for finding the value of $\sqrt{2}$, if the *Sūtrakāras* would have thought that the value of $\sqrt{2}$ derived at as per rule was correct.

(5) The conditions laid down by Vogh, for determining the value of the irrational quantity, which were found in the 17th century, can not be said to apply to the discovery of this irrational namely $\sqrt{2}$, which took place in 800 B. C. or nearly 3000 years before, when the development of mathematics was only in the primary stage.

Now I will conclude this lengthy statement by quoting Dr T. L. Heath himself as to howfar Pythagoras had any knowledge about the value of $\sqrt{2}$.

In his book " The Thirteen books of Euclid " he says :

(a) Eudemian summary in Proclus credits Pythagoras with the discovery of the irrationals.

His own comments on above :

(b) Now everything goes to show that this discovery of irrational was made with reference to $\sqrt{2}$, the ratio of the diagonal of a square and its side. It is clear that this presupposes the knowledge that the Pythagorean Theorem is true of an isosceles right-angled-triangle and the fact that some triangles of which it had been discovered to be true were right angled triangles was doubtless what sugested the enquiry whether the ratio between the lengths of the diagonal and the side of a square could also be expressed in whole numbers.

Now what is stated by him above is contradicted by T. L. Heath himself. He writes in his book " The manuel of Greek Mathematics " published in 1931 – re: Irrational on page 54.

" The first discovery of the existence of the irrational must however have been made as a result of arithmetical considerations or reasoning with numbers. It is certain that it was made with reference to the diagonal of a square in relation to its side. The first irrational to be discovered was $\sqrt{2}$. This discovery can hardly have been made by Pythagoras himself but it was certainly made in his school. The approximate date can only be conjectured ".

## CHAPTER VI

### The Theorem and its Proof

Did the Ancient Hindus posses any scientific proof of the above proposition ?

No one would expect that a scientific proof as obtained by the present standard should be available, say about 2500 years back.

In the article "The growth and development of the Theorem of Hypotenuse in India," how the Hindus approached the general proposition by stages is clearly indicated. This will clearly prove that the Ancient Hindus never noticed the truth of this important Theorem by trial only in particular cases.

Everyone agrees that the enunciation as stated by all the *Śulbakāras* is the same as that of Pythagoras.

In this article it is proposed to show how the Theorem was proved.

The principle approach to the proof was in three different ways : (1) Divide the figure and readjust. (2) Examples based on the general enunciation, and (3) Based on the number of square units in each side.

1. The following are some of the examples where the method of divide and readjust is used.

(a) To combine two unequal squares. Kāt. Śl. 2.22.
(b) To deduct a square from another square. Kāt. Sl. 3.1.
(c) To transform a rectangle into a square. Kāt. Śl. 3.2.
(d) To transform a square into a rectangle. Kāt. Śl. 3.4.
(e) To transform a triangle into a square. Kāt. Śl. 4.7.
(f) To transform an *ubhayataḥ-prauga* (two triangles) into a square. Kāt. Śl. 4.8.

It will be seen that in all the obove rules the words उपच्छेद्य; विभज्य; यथायोग्यम् उपदधानम् and उपदध्यात् etc. are invariably used meaning thereby to divide the figure and readjust the same suitably.

Let me conclude this statement by placing before you what late Prof: Naik has to say on this point: "Let us now consider the indications afforded by the *Śulba* sūtras of the knowledge of a general proof of the Theorem of the square on the hypotenuse. It may be observed that many of the constructions described in the *Śulba sūtras* carry with them their own immediately convincing proof. For the method of transforming one figure into another or compounding two figures into one employed in the *sūtras* is in general that of division into suitable parts and an appropriate readjustment of them".

"When therefore a *Sūtra* does not fully or explicitly describe its application, in any particular case, it will not be far fetched inference to take the application as implied therein".

It is more than probable that it was thus that the Hindu priests proved the truth of the Theorem and this simple and natural method, handed down from the past is found preserved practically in the same form in the work of the later writers such as *Bhāskarācārya*.

In the "History of Philosophy: Estern and Western" (published by George Allen and Unwin Ltd. London 1952) A. N. Singh writes on Geometry as follows:

"The Vedic Indians knew the mensuration of the triangle, the parallelogram, the rectangle etc. They also knew that the ratio between the circumference and the diameter of the circle is constant and tried to find out the value of this constant. There is evidence to show that they found out their results by methods which were unknown to the Greeks. The most powerful method employed by Indian Mathematicians in the field of mensuration may be called the theory of deformations which leaves areas and volumes unaltered".

"It may be mentioned here that the technique of dividing a plane figure or solid into an infinite number of parts and summing the areas or volumes of those

parts (i.e., summing an infinite series) for finding the area or volume was also used by Indian mathematicians".

2. There are a number of examples given in the *sūtras* in general form only to illustrate the enunciations: These are:

(a) The construction of the *Paitṛki-vedi*. (Kāt. Śl. 2.6).
(b) Combination of two equal squares. (Kāt. Sl. 2.12).
(c) Combination of two unequal squares. (Kāt. Śl. 2,21).
(d) Difference of two squares. (Kāt. Śl. 3.1).
(e) The threefold producer. (Kāt. Sl. 2.14).
(f) Transform a rectangle into a square and vice versa. (Kat. Śl. 3.2, 3.3),
(g) The rule illustrating, how to find the value of $\sqrt{10}$ and $\sqrt{40}$. (Kāt. Sl. 2.8 and 2.9).

3. The third method of proof was based on the number of square units in each side. This rule is stated by both *Kātyāyana* and *Āpastamba* as under:

"As many units of a measure as are in a cord, so many squares of those units in that measure, there will be in a square on that cord as a side, one should add them". Kāt. Śl. 3.7 Āpa. Śl. Page 57,

By drawing the squares actually, according to the units in each side and counting and adding them, it will be seen that the statement made is correct.

But this statement as regards the proof of the Theorem of the hypotenuse cannot be taken as correct as it takes for granted the relation between the sides and the hypotenuse in the case of a rt. ∠ed △ as granted which is still to be established. But the statement itself is a proof.

(1) That the knowledge of this relation was already known.

(2) It is not a specific instance but is stated as a general rule covering all the relations already established and to be established in future.

On this point B. B. Dutt says: "Burk surmises that the proof of the Theorem was discovered just in the figure of the *Caturasra Śyena Citi*". "The body of the *Śyena* (Ātmā) (wings and tail excluded) is a square consisting of 4 sq. *puruṣas*[1]".

HBGK = The *Ātmā* of the *śyena citi* (Fig 41).
= 4 sq. *puruṣas*,
= □ DEHA + □ DABC
÷ □ DCGF ÷ □ FKED.

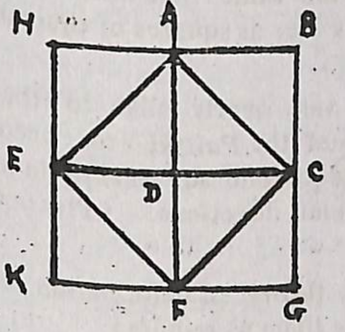

Fig 41

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[1] तस्यात्मा । समचतुरश्रश्चत्वारः पुरुषाः । बौ. शु. ३.१६.१७. सपुरुषश्चतुरश्रः । एवं प्रदक्षिणं चतुर आत्मनि पुरुषानवमिमीते । आप.शु.पृ.१४९.

each small square = 1 sq. *puruṣa.*

Now the square ACFE on the diagonal AC of the □ ABCD out of the 4□s forming the *Ātman* (or body) of this *Śyena citi* is obviously equal to the □ ADEH on AD and □ DCGE on the side DC.

Burk has further confirmed his hypothesis by a reference to *Baudhāyana's* imperfect rule (taught also by *Kātyāyana* ) for the transformation of a squares into a rectangle. See Fig. 42.

" This hypothesis about the discovery of the proof of the Theorem of the square of the diagonal of a square is endorsed also by Heath. It is certainly more likely than the one suggested by Cantor and Alman about Pythagoras's proof of the Theorem. ( See Fig. 43 ).

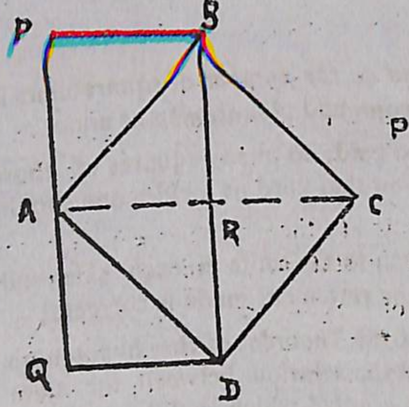

Fig 42

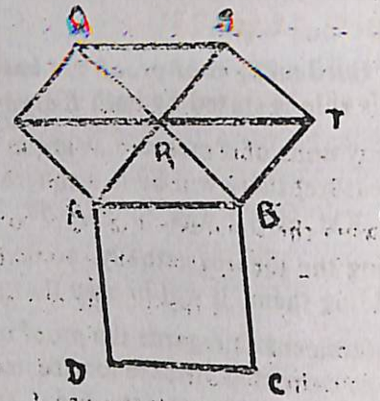

Fig 43

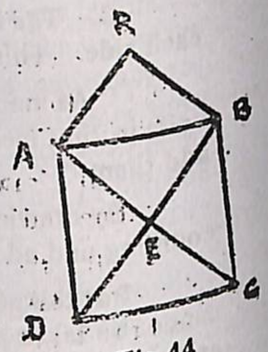

Fig 44

Dr. Thibaut says : " The authors of the *sūtras* do not give us any hint as to the way in which they found their proposition regarding the diagonal of a square, but we suppose that they too were observant of the fact that the square of the diagonal is divided by its own diagonals into 4 △s; one of which is equal to half the first square ( See Fig. 44 ). This is at the same time immediately a convincing proof of the Pythagorean proposition as far as squares of equivalent rectangular triangles are concerned.

An equally convincing hypothesis which is very nearly alike to that of Burk is suggested by the method of the construction of the *Paitṛkī* Vedi : according to one tradition about that *Vedi*, it is a square of one sq. *puruṣa* in area and whose corners are turned towards the cardinal directions. ( Please See Kāt : Śl. : 2-6 ).

The solution of this rule is obtained by the theory of deformation or by dividing the figure into several parts and readjusting them as required.

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## CHAPTER VII

### General Formulae

The following three general formulae are the most important from which any number of rational right-angled triangles can be formed. These are :

(A) $x^2 + \left(\frac{x^2-1}{2}\right) = \frac{(x+1)}{2}$ ...... Pythagoras (540 B.C.)

(B) $(2x)^2 + (x^2-1)^2 = (x^2+1)^2$ ...... Plato (380 B.C.)

(C) $(2x+1)^2 + \frac{(2x+1)^2-1}{2} = \frac{(2x+1)^2-(1+1)^2}{2}$ ...... Proclus 460 A. D.

Had the ancient Hindus any general method for finding these rectangles : Yes. They had, but this like the proof of the theorem can only be based on inference and conjecture.

These can be found out by three different methods which are based on the following constructions :—

( 1 ) Transforming a rectangle into a square. Kāt Śl. 3.2.

( 2 ) Transforming a number of equal squares into one square. *Kāt* Śl 6,7.

( 3 ) Increase of a given square into larger square (this is based on a rule given by *Āpastamba* ).

( I ) Now regarding (1) the general formulae arrived at the end of the solution of *Kāt* Śl : III-2.

$\frac{(a-b)^2}{2} + ab = \frac{(a+b)^2}{2}$ in the above substitute $a = x^2$ and $b = 1$.

we have $\frac{(x^2-1)^2}{2} + x^2 = \frac{(x^2+1)^2}{2}$ ...... A. The result is exactly similar to A quoted above, in the name of Pythagoras.

Now multiplying the result as obtained in A by 4 we get the following result which is exactly similar to the formulae derived by Plato.

$$\therefore\ 4\,\frac{(x^2-1)^2}{2} + 4x^2 = 4\,\frac{(x^2+1)^2}{2}$$

$$(x^2-1)^2 + (2x)^2 = (x^2+1)^2 \text{ ......B.}$$

And then by providing $x = 2x + 1$ we arrive at the formula derived by Proclus :—

$$[(2x+1)^2 - 1^2] + [2 \times (2x+1)^2] = [2x+1)^2 + 1]^2$$

and now dividing by 4 throughout we have :–

$$\left(\frac{(2x+1)^2-1)}{2}\right)^2+(2x+1)^2=\left(\frac{(2x+1)^2+1)}{2}\right)^2$$

or $$\left(\frac{(2x+1)^2-1)}{2}\right)^2+(2x+1)^2=\left(\frac{(2x+1)^2-1}{2}+1)\right)^2 \ldots\ldots C.$$

II. The general formulae arrived at as per *Kāt* : Śl. 6.7 is as follows :—

$$\frac{(x+1)^2}{2}\cdot a^2-\frac{(x-1)^2}{2}\cdot a^2=xa^2$$

In the above by substituting $x = x^2$ and $a = 1$. we have :

$\frac{(x^2+1)^2}{2}\cdot 1-\frac{(x^2-1)}{2}\cdot = x^2\cdot 1$. Therefore, on changing sides we have

$$\frac{(x^2-1)}{2}+x^2=\frac{(x^2+1)^2}{2}\ldots\ldots A$$

This is similar to the formula quoted above in the name of Pythagoras. Similar results can be obtained from (2) and (3).

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## APPENDIX A

### Table of Measurements

1. *Aṅgula* = 6 barley corns with their widths placed side by side or 34 seasam seeds.
2. *Puruṣa* = 120 *aṅgulas*; or 5 *aratnis* or ten *vitastis* or ten *padas*.
3. *Aratni* = 24 *aṅgulas* = *prādeśa*.
4. *Vitasti* = *pada* = 12 *aṅgulas*.
5. *Eśa* = 188 *aṅgulas*.
6. *Akṣa* = 104 *aṅgulas*.
7. *Yuga* = 86 *aṅgulas*.
8. *Śamyā* = 32 *aṅgulas*.
9. *Vyāma* = 96 *aṅgulas*.
10. *Prakrama* = This measurement varies according to the nature of sacrifice and is dependent on *Pada*.

## APPENDIX B

### Chronological Data

(a) The dates are based on the information offered by Mr. Dharma Deva Mehata in his book entitled " some positive sciences in the Vedas " published in 1959 :— He says chronology (as agreed by Western scholars) of earliest India and Greek Mathematicians.

| Sr. No. | Mathematician's Name | Name of his work | Date |
|---|---|---|---|
| 1. | *Baudhāyana* | *Śulba Sūtra* | 800 B. C. |
| 2. | *Āpastamba* | " | 400 B. C. |
| 3. | *Ārya Bhaṭṭa* | *Ārya Bhaṭṭīya* | 499 A. D. |
| 4. | *Varāha Mihira* | *Pañca Siddhāntikā* | 505 A. D, |
| 5. | *Bhāskara* 1st | *Laghu Bhāskarīya* | 522 A. D. |
| 6. | *Brahmagupta* | *Khaṇḍa Khādya* | 628 A. D. |
| 7. | *Bhāskara* 2nd | *Bīja Gaṇita* | 1350 A. D. |
| | | **Greek Mathematicians** | |
| 1. | Pythagoras | | 580 B. C. |
| 2. | Horaclidas | | 388 B. C. |
| 3. | Aristotle | | 384 B. C. |
| 4. | Pliny | | 2nd century A. D. |

(B) Chronological table as given by Dr. P. V. Kane in the history of Dharmaśastra Vol. III.

| | | |
|---|---|---|
| 1. | The period of *Vedic Saṁhitās* | 4000 B. C. to 1000 B. C. |
| 2. | *Nirukta* | 800 B. C. to 400 B. C. |
| 3. | *Śrauta sūtras* | 800 B. C. to 400 B. C. |
| 4. | *Pāṇini* | 600 B. C. to 300 B. C. |

(C) Sanskrit Literature by C. V. Vaidya.

| | | |
|---|---|---|
| 1. | *Ṛgveda* | 4000 B. C. |
| 2. | *Śatapatha Brāhmaṇa* | 3000 B. C. |

(D) In the history of Astronomy by S. B. Dixit he says that Max-Mūller gave the following dates :

| | | |
|---|---|---|
| 1. | *Vedic Saṁhitā* | 3000 B. C. to 1000 B. C. |
| 2. | *Sūtra Literature* | 800 B. C. to 600 B. C. |

All the Vedic literature was ready before the coming (advent) of *Buddha*.

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## APPENDIX C

### Progress of this Proposition in China, Egypt and Other Countries

Extract from the " Development of Mathematics " in China and Japan by Yoshio Mikami.

(1) **China :** The oldest mathematical work, left by the ancient Chinese that remains extant to this day is doubtless the Chau-Pei. The author of Chou Pei has ever remained unknown; nor is known the date when it was composed.

In the first book of Chou Pei, there is a famous dialogue that had taken place between the sage prince Chou Kong and his learned minister Shang Kao. The dialogue recorded in it serves to reveal the state of mathematical studies the Chinese had attained in so early an age as the 12th century B. C. for it was in the year 1105 that Chou Kong died. The dialogue runs as follows :

" of old Chou Kong inquired of Shang Kao : In the old times Fu hi measured the heavens susceptible of climbing, nor is the earth of measurement by means of our measures. Whence will it be then, that their numbers are deduced from ? "

Shang Kao replied : The art of numbers is derived from the circle and the square. The circle is derived from the square and the square from the Kuei or the right angled lineal; the lineal comes from 9 × 9 = 81. Thus break the lineal and make the Kau or be breadth 3, the Ku or length 4; then the distance joining the corners is 5. Take the square of the outer numbers and halving the lineal is obtained. The last sentence is very hard to be understood. It perhaps means that taking the sum of the squares on two sides of a right angled triangle and extracting its square root the hypotenuse will be obtained.

As will appear from the text in the dialogue the Chinese in the time of Chou Kong had known of the Pythagorean Theorem, so important in application that relates to the squares described on the sides of a right angled triangle. Although it is not enunciated in such a concise geometrical form as is given in Euclid, yet there is no denying the fact of its being soundly established by the Chinese in great antiquity.

The Chinese are sometimes considered as a race akin in blood to the ancient Babylonians. If the Chinese could be supposed to have borrowed their knowledge from their Babylonial Kinsmen, then why could the latter not be thought on the contrary to have learned from the former ? Pythagoras who flourished six long centuries later than Chou Kong, is he not said to have travelled in the East to Babylon and perhaps farther on probably to India ? Will it not be probable that he had seen the Theorem, that was destined to be connected with his name, on his travel ? Might he not have encountered it brought from China in some unknown way ? The lapse of time between him and Chou Kong justly makes the matter an open question.

(2) **Egypt :** (A) History of Mathematics by D. E. Smith Vol. II.

The Egyptians knew a similar relation for numerical cases, for a papyrus of the 12th dynasty (c 2000 Б. C.) discovered at Kahun refers to four of these relations one being $1^2 + (3/4)^2 = (1\frac{1}{4})^2$. It was among the people that we first hear of the "Rope stretchers" those surveyors, who it is usually thought, were able by the aid of this property to stretch a rope so as to draw a line perpendicular to another line.

(B) Extracts from the History of Mathematics by Florian Cajori : A historic papyrus, included in the collection of the British Museum was deciphered by Eisenlohr in 1877. This puts us with the mathematical thought in Egypt. It was written by Ahmes, some time before 1700 B. C. In geometry the Egyptians knew only of making constructions and determining areas. The papyrus also explains such problems as these to mark out in the field a right angled triangle a trapezium etc.

The base lines of the pyramids run North and South and East and West, but probably only the lines running North and South were determined by astronomical observations. The Egyptians like the Indians and Chinese, constructed a right angled triangle, upon a given line by stretching ropes. Thus the Egyptians were familiar before 2000 years B. C. with the well known property of the rt. ∠ed △ when the sides are 3, 4 and 5.

About 7th century B. C. an active commercial intercourse sprang up between Greece and Egypt. Greece thirsting for knowlede sought the Egyptian priests for instruction. Egyptian ideas were thus transplanted across the sea Greek culture therefore is not rather original. To Egypt, Greece is indebted among other things for elementary geometry. The Egyptians carried geometry no further than was absolutely necessary for their practical wants. The Greeks had a strong speculative tendency.

(C) Manual of Greek mathematics by Heath : The Egyptians indeed do not seem to posses it for although they knew $3^2 + 4^2 = 5^2$ there is nothing in their mathematics so far as known to us to suggest, that they knew that the triangle (3, 4 and 5) is right angled.

(3, As to the Babylonians, it was only in 1928–29, certain tablets were for the first time interpreted as containing mathematical problems. Two of which are supposed to be based in some form or another, on the property of the rt, ∠ed △. The particular case used being (12, 16, 20) which is equivalent to (3, 4, 5). Heath believes the tablets as old as 2000 B. C. and concludes that :the Babylonians of 2000 B. C. knew of the property of the square on the hypotenuse of a rt. ∠ed △ with sides in the ratio of numbers 3, 4 and 5.

India (1) Cajori says : The term *śulba sūtra* means the rules of the cord. **The *Śulba-sūtras* were composed some time between 800 B. C. and 200 A. D.** They are known to modern scholars through three quite modern manuscripts. The mathematical parts relate to the construction of squares and rectangles. Strange to say none of these geometrical constructions occur in later Hindu works.

(2) Smith in Vol—II says :

The Hindus knew the property long before the Christian era, for it is mentioned in the *Sulba sūtras* Although the date of these writings is uncertain it is evident that the relations were known rather early in India.

*Summary* : (1) According to the Western Scholars the people of (1) (1) China (2) Egypt (3) Babylonia and (4) India had the elementary knowledge of the Pythagoras Theorem.

(2) The development of this proposition was very old i.e. about 1500 to 2000 B. C. in Egypt, China and Babylonia but according to them the proposition is found in India at about 800 B. C.

(3) There appears to be a similarity of procedure of rope stretching adopted by both Egypt and India.

(4) Greece is not to be seen in this field during this period.

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## APPENDIX D

### A Short History of Pythagoras and His Works.

The date and place of his birth are both unknown. He seems to have been born between 580 to 568 B. C.. Pliny writing in the first century of our era says that Pythagoras studied in Babylon; others claim that he went as far East as India. The familiar proposition of Geometry that bears his name, was known already in India; China and Egypt before his time. He may possibly have proved the theorem—relating to the square on the hypotenuse. He died in exile (500 B. C.).

Pythagoras has left behind no mathematical treatise and the sources of information about him are very scanty. Like Egyptian Geometry. the Geometry of Pythagoras is much concerned with areas. He had probably learnt from the Egyptians the truth of the above theorem but what the Pythagorean method of proof was, has been a favourite topic of conjecture.

It is worth noting that on the circle, no theorem of any importance was discovered by his school.

The early Pythagoreans published nothing and that they referred all their discoveries back to their master. Hence it is not possible to separate his work from that of his early disciples. We must, therefore, treat the Geometry of the early Pythagorean school as a whole,

The proof of the proposition is attributed to Pythagoras by various writers but none of them lived within five centuries of Pythogoras so that we have only a weak tradition on which to rest the general belief that Pythagoras was the first to prove this theorem.

He is supposed to have discovered the irrational—with reference to $\sqrt{2}$, also.

Now let us see what other writers say about his Philosophy :—

(A) Theosophist : June 1956 pages 180 to 186.

Pythagoras 2500 years after : by J. L. Davidge.

Pythagoras is acknowledged to be the greatest scientist of antiquity, an intellectual giant and established a school of instruction which became the ancestor of all schools of idealistic society.

He went on a great tour Eastwards, travelling through many of the countries of the Mediterranean basin. With the Egyptian hicrophants he spent 22 years. being initiated at Sais; and was then carried captive under Cambyses with other sages to Babylon. After 12 years among Chaldean and Persian astronomers and magicians he journeyed to India.

In some Indian books, Pythagoras is referred to under the name of "*Yavana-Āchārya*", Ionian teacher. For about 40 years he studied the secret wisdom to the West. wisdom of the East. Then about 30 years he taught the

H. P. Blavatsky in the secret Doctrine states, that he grounded the tenets of his philosophy on the metaphysical subtleties of Buddhism, which the pytha- gorians "alone in all antiquity" understood perfectly and she quotes Ammonius Saccas, founder of the Neoplatonic school of Alexandria as teaching that Pytha- goras (like Plato) derived much of his knowledge from the books of Thoth (Hermes) and that these books were identical with the teachings of the sages of the remote East. She also relates much of his esoteric learning to the doctrines of Orphus, who according to proclus "was an Indian and came from India"

pythagoreans recognized, as scientists do to day, a fundamental relation between music and number. The mathematicians brought geometry, mathematics and music into relation with one another and worked out correrpondences between them which are very remarkable.

Now regarding the Orphic cult and his philosophical writings, let us see what others have to say about him :

(B) History of Philosophy, Eastern and Western, sponsored by the Ministry of Education, Government of India. London 1952. Introduction pages 23–24.

"The idea of salvation as the liberation of the soul from the body is a central theme in the Orphic cult. Zeller admits that this idea originated in India but nevertheless he held that the Greeks had derived it from Persia Later research does not however, indicate that such an idea of liberation or *Mokṣa* was an essential element in the Zarathustra's Faith, It would not therefore be unreason- able to suppose that this concept travelled from India to Greece and influenced the early Greek schools directly or indirectly.

It was an accepted belief in Greece that a journey to the East was necessary for the acquisition of knowledge and wisdom. It is well known that Solon and Plato had travelled extensively in the East. It would, therefore, not be surprising if Pythagoras or some other Greek philosophers of this period had travelled to India. But there is no historical evidence of such a visit. It has, however, been generally recognised that Philosophy of Pythagoras contains elements which are characteristically Indian.

We find it stated in the accounts of Alexander, that his teacher, Aristotle, had requested him to find out the state of knowledge among Indians. This, in itself, suggests that the renown of Indian wisdom had reached as far as Greece before Alexander's invasion.

(C) Let us see what A. A. Macdonell has to say about his philosophy. He writes in his book. "A history of Sanskrit literature" (written in 1900, p.421).

Turning to philosophical literature, we find that the early Greek and Indian philosophers have many points in common. Some of the leading doctrines of the Eleatics, that God and the Universe are one, that everything existing in multiplicity has no reality, that thinking and being are identical, are all to be found in the philosophy of *Upaniṣads* and *Vedānta* system, which is its outcome. Again the doctrine of Empedocles, that nothing can arise which has not existed before and nothing existing can be annihilated has its exact parallel in the Characteristic doctrine of the *Sāṅkhya* system about the eternity and indestructibility of matter.

According to Greek tradition, Thales, Empedocles Anaxagoras, Democritus and others undertook journeys to oriental countries in order to study philosophy. Hence there is at least historical possibility of the Greeks having been influenced by Indian thought through Persia.

Whatever may be the truth in the cases just mentioned the dependence Pythagoras on Indian philosophy and science certainly seems to have a high degree of probability. Almost all the doctrines ascribed to him religious, philosophical, mathematical were known in India in the 6th Century B. C. The coincidences are so numerous that their cumulative force becomes considerable. The transmigration theory, the assumption of five elements, the pythagorean Theorem in Geometry, the prohibition as to eating beans, the religio-philosophical character of the Pythagorean fraternity, and the mystical speculations of the pythagorean school, all have their close parallels in ancient India. In spite, however, of the later tradition it seems impossible that pythagoras should have made his way to India at so early a date, but he could quite well have met Indians in Persia.

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# APPENDIX E

## Explanatory Notes on Difficult Words as well as Typical performances pertaining to Various Sacrifices which are met with in the Śulba-sūtras

**Abyāsa, Abhyasya**—Having increased by itself or being doubled.

Adhvaryu-is the main priest who institutes an Adhvara and recites hymns from the Yajurveda. [*Udgātṛ* chants hymns from *Sāmaveda*, *Hotṛ* recites hymns of praise from Ṛgveda; *Brahman* keeps an overall supervision, while assisting *Adhvaryu*.]

**Agni**—means fire. In the *Śulba sūtra* it denotes an altar. The basic *citi* or the first altar is called *Sapta-Vidha*. This has an area of 7½ sq. *puruṣas*. The name of the last *citi* or *Agni* is *Ekaśatavidha*. The term *Agni* also means clay required for manufacturing bricks.

**Agni Cayana**—A mound of specific length and breadth and is constructed in five layers with baked bricks. There are various forms of this *Cayana*, The wellknown is *Śyena* or the falcon-shaped. The sacred fire is placed on this *Cayana*. Oblations of sacrificial materials and the different parts of the body of the animals are offered in this fire.

In the *Soma* sacrifice, the *Uttara vedi* is constructed on the East. It is called *Cayana* as it is built out of bricks. The names of a few *Cayanas* having different shapes are : a falcon, a wheel of the chariot, a tub, birds by name *Kaṅka*; *Suparṇa* etc. Along with the shape, the number of bricks in each layer of the *Cayana* and their dimensions were also fixed. The falcon *citi* is described in the *Taittirīya Saṁhitā* and the *Suparṇa citi* in the *Kātyāyana*. *Gàruḍa citi* is the other name for the falcon or *suparṇa*. No *Cayana* is performed in the first *Soma* sacrifice. The *Satyāṣādha Śrauta Sūtra* lays down that no *Cayana* should be performed in the *Vājapeya* and the *Sārasvata sattra*.

At the time of *Yajurveda*, *Agni Cayana* was considered equally important as the *Soma* sacrifice. In the history of sacrifices, *Cayana* is more important. The idol worship has its origin in the *Agni cayana*. The temple or religious architecture has its roots in this *Cayana*.

This is a special type of ritual performed along the *Soma* sacrifice. Five layers of 200 baked bricks each are laid one upon the other. A special mound is built having a definite shape and predetermined lengths and breadths. This brick masonry mound is called *Cayana* or *citi*. These *Cayanas* are of different shapes. the important shape being that of a falcon. The bricks have a shape of triangle; a square and a rectangle etc. In the first sacrifice 1000 bricks are used; in the second 2000 and in the third 3000 etc. The first masonry mound is constructed upto the height of the Knee. The height at the second time is upto the Knave and at the third time the height is upto the mouth. Five layers are laid on the first occasion, ten on the second and fifteen layers on the third. As the principal sacred fire is always deposited on this *Cayana* or *citi* it is sometimes called *Agni* also. On this Fire, oblations of *Soma* juice and limbs of animals are offered. Thus a mound built of bricks is called *Cayana*: *Citi* or *Agni*.

**Āgnīdhra**—One of the priests, who acts as an assistant to *Adhvaryu*. He must be acquainted with all the duties performed by the *Adhvaryu*. His duty is to protect the sacrifice. Before the start of any sacrifice, *Adhvaryu* will ask him whether evervthing is alright. On receipt of a message from him that there is no danger, the sacrifice will be started. This question and answer procedure is known as *Pratyāśrāvaṇa*. He is one of the most important priests required at all the sacrifices. In the *Soma* Sacrifice, he is provided with a special hall called *Āgnīdhrīya*. In this sacrifice, he is the first to receive the fees for the ritual amongst those present. His rank is third.

**Agnihotra**—The work of offering oblations daily to the sacred fire is called *Agnihotra*. To accept this work of *Agnihotra* as a ritual means the establishment of the three sacred fires : (1) *Gārhapatya*; (2) *Āhavanīya* and (3) *Dakṣiṇāgni*. This ceremony of establishing these fires is called *Agnyādhāna*. One who performs this ceremony is called *Āhitāgni*, who must offer oblations daily in the morning and in the evening to these sacred fires.

**Agniṣṭoma**—This is a norm of Soma sacrifices in which the Soma offering is performed during one day, *Agniṣṭoma* means praise of the fire. The sacrifice is mainly devoted to this praise. *Agniṣṭoma* is performed during the spring season.

There are in all, seven *Soma saṁsthās*. Out of this *Agniṣṭoma* sacrifice is the norm. The seven *soma saṁsthās* are : *Agniṣṭoma Atyagniṣṭoma*, *Ukthya*: *ṣoḍaśī*, *Atirātra*, *Aptoryāma* and *Vājapeya*.

*Sautrāmani* is the sacrifice when *Surā* is also used in its performance.

In addition to the above the following sacrifices are performed : those are (1) *Rājasūya*, (2) *Aśvamedha*, (3) *Ahīnas*, *Satras* etc., etc.

**Agnyādhāna** –The ceremony performed at the time of accepting the *Agnihotra* is known as *Agnyādhāna*. Two wooden blocks prepared out of dry wood from a pipal tree grown on a *Śamī* tree, form *Araṇis*. Fire is produced by friction by rubbing the blocks on one another. Depositing this fire at 3 or 5 places is the main work, to be performed at the time of *Agnyādhāna*. In this ceremony the following priests, excluding the sacrificer and his wife. are required : They are : (1) *Adhvaryu*, (2) *Brahman*, (3) *Hotṛ*. (4) *Udgātṛ* and (5) *Āgnidhra*.

*Agnyādhāna* is the name of the ceremony performed for depositing the *Gārhapatya* and other fires. One who performs this ceremony is called the Lord of the house *Gṛhapati* and hence the fire is called *Gārhapatya*. This is the main fire amongst the three. This should be kept burning during day and night and should be well protected. Some say that the shape of the altar of this fire should be a square.

**Ādhāna**—The religious ceremony performed by an individual desirous of maintaining the sacred fires perpetually at his residence. This individual is called an *Agnihotrin*. The procedure to be followed is called *Agnyādhāna*. Fire is created by the friction, by rubbing two pieces of dry wood of a Pipal tree. This Pipal tree must have its roots on a *Śamī* tree. When once such fire is created, it is deposited in different reserved places for the fire. The names of the fires are : *Gārhapatya*; *Āhavanīya*, *Dakṣiṇāgni*; *Sabhya* and *Āvasathya*.

**Āgantu**—The increased amount of the original measure.

**Āhavanīya**—A sacred fire. Oblations of materials such as ghee, boiled rice etc., are offered on this fire. Three sacred fires are maintained by an *Agnihotrin*. *Āhavanīya* altar is square in shape and is situated towards the East of *Gārhapatya* and at a distance of 8, 12 or more *prakramas*, from it. The fire from *Gārhapatya* is deposited in the place for *Āhavanīya*.

**Akṣa**—A measure of length equal to 104 *aṅgulas*.

**Aṅgula**—This is the measure of length used in the mesurements of different sacrifices.

(1) This is equal to 6 barley corns (seeds) placed side by side widthwise or (2) equal to 34 sesame seeds.

**Aparimita**—The thing which is not measured. Wherever this word is used it should be understood to mean " slightly in excess of the prescribed measure ".

**Aratni**—A measure of 24 *aṅgulas*. A hand.

**Aśvamedha**—This is a *soma* sacrifice performed on a bigger scale. *Āśvalāyana Śrauta Sūtra* says that one desirous of achieving success and prosperity should perform this. This was generally performed by Kings. In ancient times, this was a fair way of increasing the boundaries of one's state. This was considered to be of national importance leading to prosperity and at the same time it was an emblem of National unity. It was the King of all sacrifices (Śat. Br. 13.2.2.1: 13.2.9.2; Tait. Saṁ. 3.9.7.1). This was a well established sacrifice and was being performed even prior to *Ṛgveda*.

**Atirikta**—Left over.

**Cātvāla**—This is a pit dug out at a distance of 4 to 6 steps on the North side of the *Mahāvedi*. The *Uttaravedi* is prepared from this earth. Bricks are prepared from this earth and used in different *citis*. All the rubbish is then thrown in this pit.

**Cāturmāsya**—The period of 4 months from the 11th day of the first half of *Āṣāḍha* to 11th day of the 1st half of *Kārtika* or from the full moon day of *Āṣāḍha* to the full Moon day of *Kārtika*. The 11th day of *Āṣāḍha* is called *Śayanī Ekādaśī* and the 11th day of *Kārtika* is called *Bodhinī*.

**Dakṣiṇāyana**—is the night of Gods and the *Uttarāyaṇa* is the day. The *Uttarāyaṇa* is over, when the Sun enters *Karka*, and the *Dakṣiṇāyana* starts. In the month of *Āṣāḍha* the Sun enters *Karka*, hence the 11th day of *Asāḍha* is called *Śayanī* which indicates that the night of Gods is started. The night of Gods ends on the 11th day of *Kārtika*.

**Cāturmāsya** Sacrifice—This sacrifice is known for four different *parvans* These are : (1) *Vaiśvadeva*, (2) *Varuṇapraghāsa*, (3) *Sākamedha* and (4) *Śunāsīrīya*.

The duration of each *parvan* is of four months. At present all the four *Parvans* are observed simultaneously. If *Vaiśvadeva parvan* is started on the full moon day of *Caitra*; *Varuṇapraghāsa and Sākamedha* are observed from the full moon day of *Śrāvaṇa* and *Mārgaśirṣa*. The *Śunāsīrīya* can be observed at any time.

**Vaiśvadeva parvan**—This is a small *Iṣṭi*. The main deity of this *parvan* is *Viśvedevas* and hence the name. This *Iṣṭi* is performed on the three sacred fires. Eight different Deities receive oblations of different materials.

**Varuṇa praghāsa**—There are two *Iṣṭis* to be performed in this *Parvan*. The first *Iṣṭi* is performed on a bigger scale than the second one. *Marut* and *Varuṇa* are the principal deities. The other deities are similar to *Vaiśvadeva Parvan*. *Yava* grains, which are owned by *Varuṇa* and which are his favourite are used in this *parvan* and hence it is named *Varuṇa-praghāsas*. Two *Vedis* are constructed, on the east of the *Āhavanīya*. The *Vedi* on the north is in charge of the *Adhvaryu* and the *Vedi* on the south is in charge of the *Pratiprasthātṛ*. *Pratiprasthātṛ* observes and follows the same procedure which is adopted by the main priest *Adhvaryu*.

**Sākamedha**—This means igniting or kindling all the fires simultaneously. Only two days are sufficient for this *Parvan*.

**Śunāsīrīya**—*Indra* is the main deity of this *parvan*. The word *Śunāsīra* occurs in *Ṛgveda* (4.57.5). *Sunam* = Air (*Vāyu*) and *sīra Āditya* or *Sun* (Śat. Br. (2.6.3.2) or *Śuna* = prosperity. By observing this *Iṣṭi* one gets wealth, grains and cattle in plenty.

**Citi**—is a word derived from √*ci* which means to arrange or to protect *Citi* means a mound constructed of baked bricks. There are 17 *Citis* : (1) *Chandas*, (2) *Kaṅka*, (3) *Alaja*, (4) *Prauga*, (6) *Ubayataḥ-Prauga* (6) *Katha-Cakra*, (7) *Droṇa*, (8) *Samuhya*, (9) *Pariçāyya*, (10) *Smaśāna* etc. In addition to the above there are 7 *Vihavya* or *Dhiṣṇya Citis*.

*Ācārya Kaṭha* have broght into use five more Citis : These are : (1) *Sāvitra*, (2) *Nāciketa* (3) *Cāturhotra*, (4) *Vaiśvasṛja*, (5) *Āruṇaketu*. No bricks are used in their construction. Nine concentric circles are drawn and on them, small pebbles of lime stones are placed.

**Dakṣiṇāyana**—A period of six months from 21st June to 21st December, when the Sun is inclined to South or the star *Citrā*.

**Dakṣiṇāgni**—This sacred fire is situated on the South and is semi-circular in shape.

**Gārhapatya**—One of the three sacred fires. In the hall where sacred fires are placed, a circular altar of a diameter of 27 *aṅgulas* is provided with two pedestals. The bottom pedestal is of six fingers in height and breadth. The top

pedestal is four fingers in height. This circular altar is called *Gārhapatyāyatana*. The sacred fire is deposited in this with due ceremonies.

**Halls**—The following halls are constructed at the time of *Soma* sacrifice.

**Prāgvaṁśa**—This is a hall, the ridge of which is provided by a bamboo, the end of the ridge faces East. This hall is erected first. The length of the hall (East-West side) is 16 *Prakramas* and width 12 *Prakramas*. A door is provided at the centre of each side. The width of the door is 24 *aṅgulas*. In the North-East corner one more door is provided. The columns on the East are a bit taller. The sides of the hall are closed by thatch-work.

**Sadas**—This hall is a meeting place. It is erected on the West side of the *Havirdhāna* hall. It is 27 *aratnis* in length and 9 *aratnis* in width. Some keep these dimensions as 18 × 9 *aratnis*. There are two doors one on the East and the other on the West. 9 bamboo rafters, whose ends are towards the North are provided. The column on the East and West are shorter than the centre. Thus the hall slopes on both sides.

**Havirdhāna**—This hall is provided for storing materials, required for the sacrifice. It has a square shape. The length of its side is 10 hands. Two doors, one on the East and the other on the West are provided. The column adjacent to the door on the East is covered with *darbha* grass.

**Āgnīdhrīya** and **Mārjālīya**—These two halls are constructed on the North and South side of the *Mahāvedi*. The *Āgnīdhrīya* is on the North and the *Mārjālīya* is on the South. Half of the Hall is inside and the other half outside of the *Mahāvedi*. They are square in shape and the length of each side is 120 *aṅgulas*. There is a door on the South for *Āgnīdhrīya* and a door on the North for *Mārjālīya*.

**Īṣā**—The pole of a chariot. A measure of length of 188 *aṅgulas*.

**Iṣṭakā**—A baked brick made out of clay. These bricks of different shapes and sizes were used for the construction of altars.

**Iṣṭi**—On the first day of every half (dark and bright) of each month, *Agnihotrin* performs this ceremony; since the acceptance of his vow, till his death. There are different types of *Iṣṭis* to be performed; *Darśa* and *pūraṇamāsa-Iṣṭis* are performed on New and Full Moon day. *Āgrayaneṣṭi* is performed occasionally. To achieve some desired object *Vardhāpaneṣṭi* or *Pavitreṣṭi* is performed. The following *Iṣṭis* are performed in *Āgniṣṭoma* :*Dikṣaṇīyā*, *Prāyaṇīyā*. *Ātithyā*, *Avabhṛteṣṭi*, *Udayanīyā* etc. *Cāturmāsya* sacrifice consists of a number of *Iṣṭis*.

**Darśapūrṇamāsa**—The *Iṣṭi* which is performed on the following day of the Full moon is called *Paurṇamāseṣṭi* and the *Iṣṭi* performed on New Moon day is called *Darśeṣṭi*. The word *Darśapurṇamāsa* represents both of them. Oblations are offered to *Agni* and *Indra--Agni* in the *Darśeṣṭi* and to the *Agni* and *Agni-Soma* in the *Iṣṭi* in the Full moon.

**Kaṅka**—A kind of bird. Altars having shapes of birds were prepared. There were similar other altars whose names are : *Alaja. Śyena, Suparṇa* etc.

**Karaṇī**—A side of a square. This also denotes an irrational.

**Śulba** - A measuring cord or a measuring stick. *Darbha* grass or small twigs of different trees (*Samidhs*) are procured by the main priest *Adhvaryu*. By twisting the *Darbha* grass in two layers first and then adding one more layer to the same a measuring cord is prepared of exact measured length, including slings at both ends. The cord which is used only for binding purposes is called *Raśanā* or *Rajju*.

**Udagayana**—The period of six months from 21st December to 21st June, when the Sun is inclined towards the Star *Svātī*.

**Utkara**—This is a place where all the grass and earth removed from the surface, while levelling the ground is dumped.This is at a distance of two *padas* and on the north side of the *Mahāvedi*.

**Uttara-Vedi**—An altar constructed on the East side of the sacrificial place.

**Vedi**—is a general term which denotes the whole area required for the performance of the sacrifice.

**Dārśikī vedi**—The length of the East-west line is 96 *aṅgulas*, prepared at the time of *Darśapūrṇamāsa*. Its width is limited by utensils required for the sacrifice. All these utensils must remain in this area. The sides on the North and south are slightly bent inside. The sides on the east and west are straight. The space occupied by the Vedi is dug upto a depth of 4 *aṅgulas* and the pit so formed is then filled up with dry earth.

**Pāśukī-vedi**—The East-West length of the *Vedi* provided for the animal sacrifice is 188 *aṅgulas*. Some prefer to adopt the following dimensions. East-west line-6 *aratnis* or 144 *aṅgulas*. East side=3 *aratnis* and West side = 4 *aratnis*.

A Vedi with an oblong shape and 36 *aṅgulas* in height is constructed inside. The *Adhvaryu* digs out earth from *Cātvāla* and prepares a big mound of earth on this a small mound of the size of a foot of a cow is prepared. This small mound is called *Nābhi*.

**Saumikī vedi**—This is a vedi constructed for the *Soma* sacrifice. The dimensions of the *Vedi* are :

East-West line 36 *prakramas*; east side 24 *prakramas* and the west side 30 *prakramas*. Inside this *Vedi*, there is a *Uttara-vedi* which is 10 Sq, *padas* on east side = 8 *padas* and west side = 12 *padas*. The *saumikī Vedi* is also called the *Mahāvedi*.

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(*b*) *Baudhāyana Śulba Sūtra* (Translation with notes by Dr. G.Thibau, published in the "Pandit" magazine; a monthly Journal of the Banaras College devoted to Sanskrit literature, May 1st 1875 to Oct. 2nd 1876).
(*c*) *Kātyāyana Śulba Śūtra*—(The Kāsi Sanskrit series No. 130).
(*d*) ,, ,, ,, (The Chokhambā Sansdrit series).
(36) Dr. G. Thibaut—On the *Śu'lba Sūtras* (published in the journal of the Asiatic society of Bengal Vol. 44, Part I-1875).
(37) Mr. Frand Thilly—History of Philosophy.
(38) Shri C. V. Vaidya—Epic India:
(39) Dr. Albrecht Weber—*Kātyāyana Śrauta Sūtra.*
(40) Mr. Will Durant—The Life of Greece.
(41) Mr. Will Durant—The story of Philosophy.
(42) Mr. Winternitz—History of Indian literature Vol. I.
(43): Mr. Yoshio Nikani—The development of Mathematics in China and Japan.
(44) Ancient Indian Mathematics—Articles from *Bhārata Jyoti* 13-5-62, 20-5-62, 27-5-62.
(45) Indian Master minds laid Foundations of Modern Mathematics. (Articles from time of India : by Ikbal Kaul) 11-3-62, 18-3-62.
(46) The Sanskrit Language—Articles from *Bhārata Jyoti* 8-1-77.
(47) *Bhāratīya Sanskṛti Kośa* (Marathi) Vols. I, II and III.
(48) Encyclopaedia Britanica 9th edition Vol. XV.
(49) *Sṛṣṭijñāna* Magazine (Marāthi)—August November, issues 1964.
(50) *Śatapatha Brāhmaṇa*—East and West series Vol. VII Part I, Volume. XXVI, Part II.
(51) *Taittirīya Brāhmaṇa.*
(52) *Taittirīyá Saṁhitā.*
(53) *Ṛg-veda.*
(54) History of philosophy Eastern and Western: Ministry of educatio Government of India.

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# Opinions

*From Learned Scholars*

I have carefully gone through the type-script of the Kātyāyana Śulba Sūtra by my friend, Shri. S. D. Khadilkar. He has brought together the several relevant passages from these ancient texts with the help of commentaries thereon and translations and monographs on them by Western Scholars.

He is uptodate in his treatment of the subject and has rendered the Sanskrit texts in simple correct English, which is indeed a boon to modern readers.

I have suggested certain changes which the writer is free either, to accept or reject.

The main point I want to emphasize in this connection is that such monographs must be immediately published and attention should be drawn to the rich but yet untapped and even undiscovered treasures of the wisdom of our fore-fathers.

I would urge the author to pursue his comparative studies in Ancient Hindu Mathematics, along with Chinese and Babilonian efforts in this field so that the civilisations of the past would lay their secrets before us and would make us realise that it is man as a whole whether in the East or West, who is in search of the unknown both within and without.

I wish the author every success in his under taking.

**V.P.Limaye**
10-5-1969 Poona 2

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I have gone through your interesting articles and I am indeed, very happy to know that you are engaged in doing important research work in history of Mathematics.

**U.N.Singh**
Professor and Head of the
Department of Mathematics,
15-9-1968 M.S. University, Baroda.

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THE SULBASUTRAS written by Shri. S.D. Khadlkar is a monumental work. None of us, excepting some European Scholars attempted on this topic. It exhaustively deals with and expounds the Geometrical datas as found in Baudhayana, Apastamba and Katyayana Sulba Sutras. The theme as Pythagoras being the first exponent of Geometry has been questioned by the learned author. Baudhayana flourished much before Pythagoras. Even Pythagoras visited India according to some authorities. He himself is indebted to the learning of this country. Some intricate words also have been explained by the author. I congratulate him on this meritorious work.

**M.R.Sharma**
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13-4-1968 Kanpur